ELEMENTS OF HEAT TRANSFER AND INSULATION

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AND
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BY

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PREFACE

" We are frequently told that heat transfer is a field for graduate tudy; that it is too difficult a subject in its rational form for the under-;raduate student. If we accept this view, we may well be accused of ccepting mechanical engineering, in the majority of its practices, as an mpirical art."

Approximately three years before Dean Wilkinson published the bove statement* the authors had begun to teach regular elective ourses in heat transfer, designed for senior mechanical and chemical ngineering students, following the general lines of the present book. 'he subjects of converting and conveying heat energy are of similar nportance. The first deals with thermodynamics and is included in lmost every engineering curriculum; the latter is now beginning to be oked upon favorably for inclusion in the undergraduate plan of study.

It, therefore, seems timely to present a textbook for courses in heat ansmission, not designed for graduate level nor exhaustive in scope, ut adapted to the average junior or senior engineering student. If no pecial course in heat transmission is offered, this text may serve as an dditional brief book in the usual thermodynamics course. In this ay the student will be given a more nearly complete picture of the ientific fundamentals of heat engineering.

In order to be suited for these purposes the book is restricted to the w basic principles of heat transfer and insulation, and to their application to simple problems. The various subjects are presented in a road manner with examples following the derivations.

However, scientific rigor has been retained to as great an extent as ossible at the intended level of presentation. This applies especially the use of consistent units, which are so important in the science and pplication of heat transfer. In this matter no concessions to superficial ractices have been made. The student is induced to insure in each pecial case that the balance of physical dimensions is correct, and articularly to avoid the usual confusion of mass and force units.

Conduction, convection, and radiation are treated separately first, d then in their combinations. A whole chapter is devoted to the asic property of thermal conductivity with particular attention to

* F. L. Wilkinson, Jr., " A Suggested Design for Mechanical Engineering Curric-a." *Mechanical Engineering*, Vol. 63, p. 581 (1941).

practical insulations. Unsteady state equations are made palatable to the student by some graphical representations. Dimensional analysis is dealt with in the most elementary manner. Some applications of heat transfer in experimental engineering and the interdependence of heat transfer and surface friction form the subject of the last three chapters.

In order to facilitate the solution of the problems, the Contents is supplemented by a list of tables and a list of figures needed for numerical calculations. These are followed by a list of symbols. Answers to the problems will be printed separately.

Space and purpose of the book forbid ample literature references. Only a very small selection of them is given at the end of the chapters. However, every effort has been made to give credit to individuals for use of their materials. Special thanks is due to Mr. H. E. Hollensbe, Editor of *Industrial Power*, for the release of certain copyrighted material herein contained in the form of problems.

The authors wish to express appreciation to President H. T. Heald of Illinois Institute of Technology, and Dr. A. A. Potter, Dean of the Schools of Engineering, Purdue University, who by initiating undergraduate courses in heat transfer, encouraged our task, and to Professor H. L. Solberg, Head of the School of Mechanical Engineering, Purdue University, for his keen interest during the preparation of this textbook.

Chicago, Illinois
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CONTENTS

TABLES

FIGURES NEEDED FOR NUMERICAL CALCULATIONS

SYMBOLS FOR PHYSICAL QUANTITIES

Symbol	Quantity	Units*
A	Area	ft^2
B	Buoyancy	$lb\ ft^{-3}$
C	Circumference; mean coil diameter	ft
c	Specific heat	$B\ slug^{-1}\ F^{-1}$
C_p	Specific heat at constant pressure related to unit volume	$B\ ft^{-3}\ F^{-1}$
c_p	Specific heat at constant pressure	$B\ slug^{-1}\ F^{-1}$
D	Diameter	ft
E	Density of emission	$B\ hr^{-1}\ ft^{-2}$
E	Electrical potential	volt
f	Friction factor	
F_A	Configuration factor or area factor	
F_E	Emissivity factor	
F_s	Shearing force (friction)	lb
g	Gravitational acceleration	$ft\ hr^{-2}$
(Gr)	Grashof number	
H	Fundamental unit of heat energy	B
h	Heat transfer coefficient	$B\ hr^{-1}\ ft^{-2}\ F^{-1}$
I	Intensity of radiation	$B\ hr^{-1}\ ft^{-3}$
I	Electric current	ampere
k	Thermal conductivity	$B\ hr^{-1}\ ft^{-1}\ F^{-1}$
L	Length, Fundamental unit of length	ft
l	Distance	ft
l	Latent heat	$B\ slug^{-1}$
M	Fundamental unit of mass	slug
M	Mass velocity	$slug\ hr^{-1}\ ft^{-2}$
m	Rate of mass flow	$slug\ hr^{-1}$
n	Frequency	hr^{-1}
(Nu)	Nusselt number	
p	Pressure	$lb\ ft^{-2}$
(Pr)	Prandtl number	
q	Rate of heat flow	$B\ hr^{-1}$
R	Thermal resistance	$B^{-1}\ hr\ F$
R	Electrical resistance (only used with prime sign or subscripts)	ohm
r	Radial distance	ft
(Re)	Reynolds number	
S	Side length of a square	ft

* For lengths, forces, and masses the units of the British gravitational system are used. Time is given in hours instead of seconds in accordance with the customary engineering practice in heat transfer work. Temperatures are given in degrees Fahrenheit (F), absolute temperature in degrees Rankine (R).

Symbol	Quantity	Units*
T	Absolute temperature	R
T	Fundamental unit of time	hr
t	Temperature	F
U	Overall heat transfer coefficient	B hr^{-1} ft^{-}
v	Velocity	ft hr^{-1}
x	Distance; thickness	ft
y	Distance	ft
α	Thermal diffusivity	ft^2 hr^{-1}
α	Absorptivity for radiation	
β	Coefficient of cubical expansion	R^{-1}
γ	Specific weight	lb ft^{-3}
$(\Delta t)_m$	Log mean temperature difference	F
ϵ	Emissivity	
ϵ	Temperature coefficient of electrical resistance	F^{-1}
Θ	Fundamental unit of temperature	F
θ	Temperature difference	F
κ	Temperature coefficient of thermal conductivity	F^{-1}
λ	Wavelength	ft
μ	Dynamic viscosity	lb ft^{-2} hr
ν	Kinematic viscosity	ft^2 hr^{-1}
ρ	Density	slug ft^{-3}
ρ	Reflectivity for radiation	
σ	Constant in Stefan-Boltzmann's law	B hr^{-1} ft^{-2}
τ	Time	hr
τ	Transmissivity for radiation	

* See footnote page xv.

CHAPTER I

INTRODUCTION

I–1 Importance of Heat Transfer and Insulation in Engineering

The laws which govern heat transmission are very important to the engineer in the design, construction, testing, and operation of heat exchange apparatus. Heat transfer problems confront investigators in nearly every branch of engineering.

Electrical engineers apply their knowledge of heat transfer to the design of cooling systems for motors, generators, and transformers. Chemical engineers are concerned with the evaporation, condensation, heating, and cooling of fluids. The mechanical engineer deals with problems of heat transfer in the fields of internal combustion engines, steam generation, refrigeration, and heating and ventilating. An understanding of the laws of the flow of heat is important to the civil engineer in the construction of dams and structures, and to the architect in the design of buildings.

In the latter instance the question of effective heat insulation will prevail. However, problems involving the use of insulation likewise occur in each of the fields of engineering. Actually every engineer will be confronted from time to time with the question of how to transmit heat in the most effective way, or of how to protect a construction most efficiently against heat or cold losses.

I–2 The Three Kinds of Heat Transfer

There are three different types of heat transfer: conduction, convection, and radiation. (Ref. I–1.) They have in common that temperature differences must exist and that heat is always transferred in the direction of decreasing temperature. On the other hand they differ entirely in the physical mechanisms and laws by which they are governed.

Heat conduction is due to the property of matter which allows the passage of heat energy, even if a physical body is impermeable to any kind of rays and its parts are not in motion relative to one another.

Heat convection is due to the faculty of moving matter to carry heat energy such as transporting a load from one place to another.

Heat radiation is due to the property of matter to emit and to absorb different kinds of rays, and to the fact that an empty space is perfectly permeable to rays and that matter allows them to pass more or less.

If the flow of heat is independent of time, it is spoken of as steady or stationary state. Engineering problems of steady flow of heat by conduction include the flow of heat through furnace walls and pipe insulation where the surface temperatures are maintained almost constant, and in general where changes of conductive heat flow with time are negligibly small.

The fundamental relation for the steady flow of heat by conduction originates from the French physicists Biot and Fourier and can be expressed by

$$q_k = kA \frac{\Delta_x t}{\Delta x} \tag{I-1}$$

This formula is based on the assumption of a homogeneous substance in which a constant temperature difference $\Delta_x t$ is held between the points of a plane area A and any points at a short perpendicular distance Δx from this area. Then a steady heat flow q_k per unit time (rate of heat flow) occurs in the direction of decreasing temperature. The factor of proportionality k is called " heat conductivity " or " thermal conductivity."

Whenever heat flows by conduction in the transient or unsteady state the temperature of a fixed point within the material does not remain constant; that is, the temperature within a material undergoing cooling or heating varies with the time. Some of the industrial problems which involve this particular type of heat transmission are: the annealing of castings, the vulcanizing of rubber, and the heating or cooling of the walls of buildings, furnaces, and ovens.

In unidirectional flow of heat by conduction in the transient state, in addition to Eq. I-1 the following relation for the rate of heat stored q_{st} has to be considered:

$$q_{st} = \rho c_p (A \cdot \Delta x) \frac{\quad}{\Delta \tau}$$

Here $\Delta_\tau t$ means the average temperature increase of the substance in the small volume $A \cdot \Delta x$ in a short time interval $\Delta\tau$, when ρ and c_p are the density and specific heat, respectively, of that substance. $\rho c_p (A \cdot \Delta x)\Delta_\tau t$ apparently is the heat energy stored in the volume $A \cdot \Delta x$ in the time $\Delta\tau$.

Heat transfer by convection occurs on walls of rooms, on the outside of warm and cold pipes, and between the surfaces and fluids of all of heat exchangers.

For this type of heat transmission the following equation which goes back to Newton, is in general use:

$$q_h = hA \cdot \Delta t \qquad [I-3]$$

It simply states that an invariable temperature difference Δt between the area A of a surface and a fluid in contact with it, causes a steady heat flow q_h. The factor of proportionality h, defined by this equation, is called " film coefficient of heat transfer " or simply " coefficient of heat transfer."

Heat transfer by radiation is important in boiler furnaces, billet-reheating furnaces, and other types of heat exchangers. Solar radiation plays an important part in the design of heating and ventilating systems.

The rate of heat, q_r, radiated from an area, A, is given by the equation

$$q_r = \epsilon\sigma A T^4 \qquad [I-4]$$

in which T is the absolute temperature, ϵ is a factor depending on the kind of surface and on temperature, and σ is a constant of nature, independent of both surface and temperature. It will be shown that for a perfectly black surface $\epsilon = 1$. In this case Eq. I–4 simplifies into Stefan-Boltzmann's law, so called in honor of Stefan who found it empirically and Boltzmann who proved it theoretically.

It is important to note that the majority of industrial problems dealing with heat exchangers do not involve a single mechanism of heat transfer, but a combination of two or more. In a steam condenser the transfer of heat is both by conduction and convection from the condensing steam to the cooling water. In the furnaces of large steam generators heat is transferred by radiation, convection, and conduction.

In the following chapters the basic equations governing each type of heat transfer and cases dealing with a combination of two or more of the ways of heat transfer will be considered.

I–3 Basic Units

Consistency of units in numerical calculations is even more important in heat transfer than in most other fields of engineering. This is due to the complex arrangement and combination of units in equations. For instance, many formulas are given as products of power functions of the variables involved which often have fractional exponents. There are some texts and articles of good standing which contain equations and tables of mixed units, the use of which leads to serious errors unless in each case careful individual consideration is given to the conversion of units.

As usual in engineering calculations on heat transmission, in this text foot and hour are generally used as units of length and time. In

addition the use of the units degree Fahrenheit and British thermal unit is obvious. The main difficulties arise in the use of the pound force and the corresponding unit of mass, the slug. The latter is that mass which when acted upon by a force of one pound will be accelerated 1 ft/sec^2.

Sometimes, however, the pound mass will be used as a unit. This will be denoted by lb_m, whereas lb without subscript or with the subscript f means pound force in this text.

Confusion in the use of force and mass units must be avoided first of all in dealing with the density, ρ, and the specific weight, γ. Because the numerical value of the former in [lb_m/cu ft] is equal to that of the latter in [lb_f/cu ft], they often are considered as identical. This however is erroneous. In the *same* system of units they differ by the gravitational acceleration g as a factor according to the equation

$$\gamma = \rho \cdot g \qquad \text{[I–5]}$$

The unit of density in the foot-hour-pound force system is the slug per cubic foot. According to Eq. I–5 the specific weight which is expressed in pound force per cubic foot may be converted to the density unit of [slug/cu ft] by dividing by 32.16 ft/sec^2. So, the density of water at 68 F is 62.305/32.16 = 1.937 slug/cu ft.

Other physical properties in which force or mass is involved are the specific heat and the viscosity. The units of these will be discussed in Sects. IV–1 and VI–2, respectively.

The physical centimeter-gram-second system and some other systems are used only occasionally to show conversion or in practice problems.

REFERENCE

I–1. M. Jakob, "A Survey of the Science of Heat Transmission," Research Series 68, *Engineering Experiment Station Bulletin*, Purdue University, Vol. XXIII, No. 4a, 1939.

CHAPTER II

THERMAL CONDUCTIVITY

II–1 Dimensions of Thermal Conductivity

In Eq. I–1 the subscript k was used to distinguish the heat flow by thermal conduction from that by convection and radiation, and x was used to distinguish the temperature difference in the direction x from the temperature change in time, denoted by subscript τ in Eq. I–2.

If only heat flow by conduction independent of time is considered, then Eq. I–1 may be written without any subscripts:

$$q = kA \frac{\Delta t}{\Delta x} \qquad \text{[II–1]}$$

The factor of proportionality k in this relation represents a physical property of the substance through which heat is conducted and is called "heat conductivity" or "thermal conductivity," as already mentioned.

When physical equations like Eq. II–1 are used it is imperative that all the constituent parts be expressed in consistent physical units. The most satisfactory way to check the physical dimensions in an equation is to assume that the numerical magnitude of each term is unity and then to consider only the remaining units. The units may be operated on as though they were the symbols of an algebraic equation. Writing the units in square brackets and expressing the physical dimensions of k one obtains* from Eq. II–1:

$$1[\mathrm{B/hr}] = 1[k]\, 1[\mathrm{sq\ ft}] \frac{1[\mathrm{F}]}{1[\mathrm{ft}]} \qquad \text{[II–2]}$$

or

$$[k] = [\mathrm{B\ hr^{-1}\ ft^{-1}\ F^{-1}}] \qquad \text{[II–3]}$$

Equation II–3 gives the physical dimensions of the property k in British technical units.

It is important to note that, whereas in Eq. II–2 sq ft and ft were used for the area $A = 1$ and the distance $\Delta x = 1$, respectively, these

* B means "British thermal unit" and F means "degree Fahrenheit," according to the "American Standard Abbreviations for Scientific and Engineering Terms," 1941.

are combined into ft^{-1} in Eq. II–3. This simplification is due to the use of consistent units, namely, sq ft for areas and ft for distances in the case considered.*

If another system of units is to be employed, it is only necessary to replace the units in Eq. II–3 by their equivalents in the other units. For example:

1 B = 252 cal (abbreviation for physical calorie†)
1 hr = 3600 sec
1 ft = 30.48 cm
1 F = $(\frac{5}{9})$ C (abbreviation for degree centigrade)

and by substitution in Eq. II–3:

$$1[\mathrm{B\ hr^{-1}\ ft^{-1}\ F^{-1}}] = \left[252\ \mathrm{cal}\ \frac{1}{3600\ \mathrm{sec}}\ \frac{1}{30.48\ \mathrm{cm}}\ \frac{1}{(\frac{5}{9})\,\mathrm{C}}\right]$$
$$= 0.00413\ [\mathrm{cal\ sec^{-1}\ cm^{-1}\ C^{-1}}]$$

This is the conversion factor of the thermal conductivity from British technical units into the usual physical units. By similar procedure, it is possible to find the following conversion factors:

$$1\ \mathrm{B\ hr^{-1}\ ft^{-1}\ F^{-1}} = 1.488\ \mathrm{kcal\ hr^{-1}\ m^{-1}\ C^{-1}}\ \text{(metric technical units)}$$
$$= 0.0173\ \mathrm{watt\ cm^{-1}\ C^{-1}}\ \text{(electro-physical units)}$$

II–2 Factors Which Influence Conductivity

The numerical values of k for different substances vary from almost zero for gases under extreme vacuum conditions to about 7000 B hr^{-1} ft^{-1} F^{-1}, which has been observed for a natural copper crystal at the very low temperature of −422 F. The values of k for a substance depend on the chemical composition, the physical state and texture, and the temperature and pressure.

From the viewpoint of molecular physics thermal conduction is considered as a transportation of heat energy due to molecular motion, whereas heat transmission connected with the ordinary flow of a fluid belongs to thermal convection.

The molecular transmission of heat is smallest in gases. It may be compared with the transportation of energy by billiard balls. In a gas space, as on a billiard table, the efficiency of the exchange of energy is excellent, but the field is almost empty, compared with the size of the

* Concerning the use of sq ft for the area and inches for the thickness of a plate see Sect. III–2.

† 1 cal is the heat quantity required for heating 1 gram of water by 1 C, 1 kcal (kilo-calorie) is the heat quantity required for heating 1 kilogram of water by 1 C.

balls, and since the balls move about in a zigzag manner the total energy exchange for unit space in a given direction is small. Thus the heat conductivity of gases is lower than for any other substances. In liquids and in electrically insulating solid bodies the heat energy is considered to be handed down from places of higher temperature to those of lower temperature by elastic oscillations similar to the propagation of sound. Because for both kinds of substances matter is packed much more closely in a given space than for gases, the heat conduction of liquids and electrical insulators is much better than that for gases. However, the oscillatory transportation of energy is hampered by the irregular arrangement of the molecules and atoms in liquids and in so-called glassy or amorphous solid substances. In contrast to this, the crystalline structure of a substance, comparable with a chain of springs, affords a much more effective propagation of heat energy in the direction of an existing temperature decrease, and therefore crystalline substances are much better heat conductors than liquids or glassy bodies. In the metallic substances, other important carriers of energy enter into the action, which are known as free electrons. Experience and theory have shown that the thermal and electrical conductivity of pure metals at the same temperature are approximately proportional.

Details of this complex picture, particularly the theoretical consideration of the influence of temperature, are beyond the scope of this text. Only two items of practical significance will be mentioned. In solids used for insulating materials, the insulating effect is due mainly to the air in the pores which as a gas is a poor conductor of heat. The apparent density of a porous or fibrous body which is defined as the ratio of its total mass to its total volume,* is small because of the large number of air voids. This accounts for the decrease in the heat conductivity of insulating materials as the apparent density becomes smaller. If water penetrates into a porous substance and fills the pores, for instance due to excessive atmospheric humidity, then the body can no longer be considered as a combination of solid material and air, but a part of it is water with a corresponding higher thermal conductivity value. If instead of fine pores, wide air spaces are considered, it might be concluded that the insulating effect would be the same as for a substance having small pores and the same apparent density. This is not true, because of air circulation within the large free spaces which decreases the insulating effectiveness of the substance. In addition, for the same temperature range, more heat is radiated through a substance

* In the tables of this chapter "apparent specific weights" are used instead of "apparent densities." See the remark in Sect. I–3.

composed of wide spaces than through the same volume with narrow spaces. This fact can easily be derived from the laws of radiation. Radiation through the pores is generally included in the value for k, as is convection if the pores are wide enough so that this, too, must be considered.

These theoretical remarks may be sufficient for a general understanding of the large variations in the heat conductivity values of various materials which are presented in the following sections for information and use in numerical calculations. The sequence is, as far as possible, from smaller to larger values of k. Where the physical dimensions are not indicated explicitly, the dimensions for k are B hr^{-1} ft^{-1} F^{-1}. Temperatures are in degrees Fahrenheit.

II–3 Gases and Vapors

Table II–1 shows the thermal conductivity at 32 F and 212 F for a number of gases and vapors along with the molecular weight. The table relates to a range in which, according to theory and experience, the pressure has no appreciable influence on the thermal conductivity. This is the case for moderate vacuum conditions, and also for approximately atmospheric pressure, except for those substances which are liquids or not far removed from saturation at this pressure. From the table, it is apparent that the thermal conductivity of gases or vapors with higher molecular weight is smaller than that for the lighter ones.

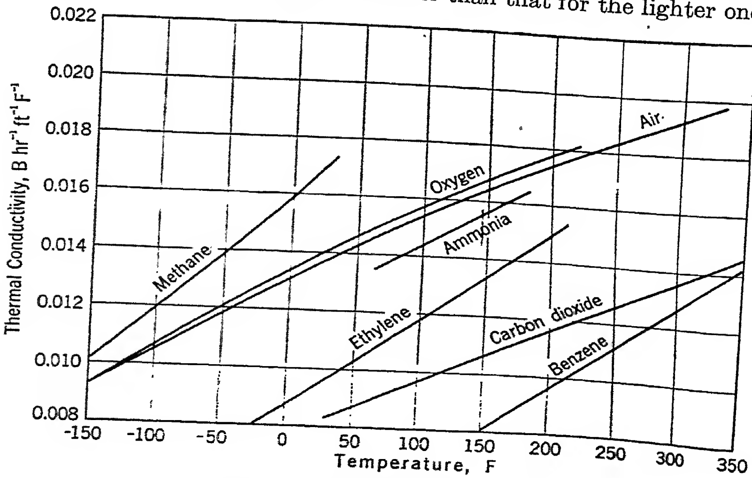


FIG. II–1. Thermal conductivity of some gases and vapors.

Figure II-1 shows the variation of the thermal conductivity with temperature for some gases and vapors. In general, the thermal conductivity increases with an increase in temperature.

TABLE II-1

THERMAL CONDUCTIVITY OF GASES AND VAPORS AT MODERATE VACUUM PRESSURES

Gas or Vapor	Chemical Formula	Molecular Weight	Thermal Conductivity [B hr^{-1} ft^{-1} F^{-1}] at 32 F	212 F
Hydrogen	H_2	2	0.099	0.124
Helium	He	4	0.082	0.097
Methane	CH_4	16	0.0175	......
Ammonia	NH_3	17	0.0124	0.0171
Water vapor	H_2O	18	......	0.013
Nitrogen	N_2	28	0.0141	0.0175
Ethylene	C_2H_4	28	0.0097	0.0154
Air	......	29	0.0141	0.0177
Ethane	C_2H_6	30	0.0104	0.0184
Oxygen	O_2	32	0.0141	0.0181
Carbon dioxide	CO_2	44	0.0081	0.0121
Ethyl alcohol	C_2H_5OH	46	0.0081	0.0121
Benzene	C_6H_6	78	0.0050	0.0101
Carbon tetrachloride	CCl_4	154	......	0.0050

II-4 Liquids

Accurate data are only available for a few liquids. Table II-2 and Fig. II-2 show the thermal conductivity of water as a function of temperature. It is seen that a maximum occurs at about 250 F. Water is the best conductor of all non-metallic liquids. The heat conductivity of organic liquids at ordinary temperature is of the order of magnitude of 0.1 B hr^{-1} ft^{-1} F^{-1} with variations of seldom more than ±20 per cent.

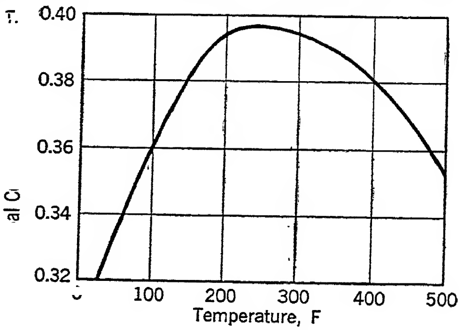


FIG. II-2. Thermal conductivity of water.

TABLE II–2

THERMAL CONDUCTIVITY OF WATER FOR VARIOUS TEMPERATURES

Temperature F	Thermal Conductivity* B hr^{-1} ft^{-1} F^{-1}
40	0.327
50	0.333
60	0.336
70	0.344
80	0.349
90	0.354
100	0.360
110	0.364
120	0.366
130	0.371
140	0.377
150	0.381

* Data taken from A. Eucken and M. Jakob, *Der Chemie-Ingenieur*, Vol. I, Part 1, Chapter VI. Akademische Verlagsgesellschaft, Leipzig, 1933.

II–5 Insulating Materials

Several of the most used types will be described here in some detail. In the selection of a suitable insulation such factors as insulation efficiency,† resistance against temperature, shock, or corrosive chemical fumes, cost, and reuse value should all be considered. In general, all heat-insulating materials may be grouped into the following general classes.

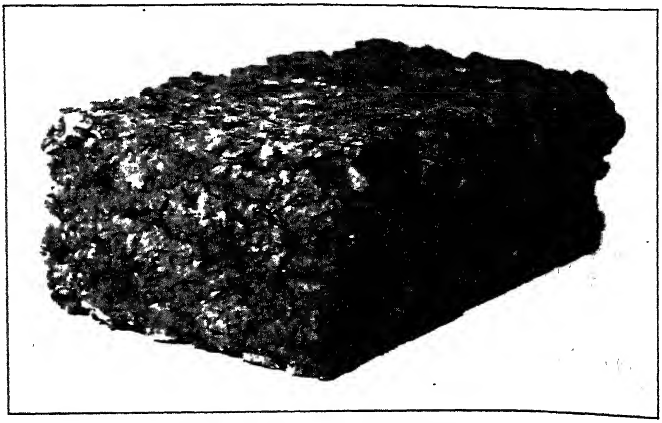


FIG. . Cork block insulation.

† Defined in Sect. XII–3.

(a) Low-Temperature Insulations

In this class the insulators are used to prevent freezing or sweating and to insulate ice-water pipes and other refrigeration equipment. The materials are manufactured so as to have a low conductivity and a durable structure under moisture conditions. The problem of keeping moisture away from the cold surfaces and at the same time keeping it outside of the insulation itself is of paramount importance in the design of low-temperature insulation. Cattle hair, wool felt, cork, and various combinations of these serve extensively as raw materials. Cork is

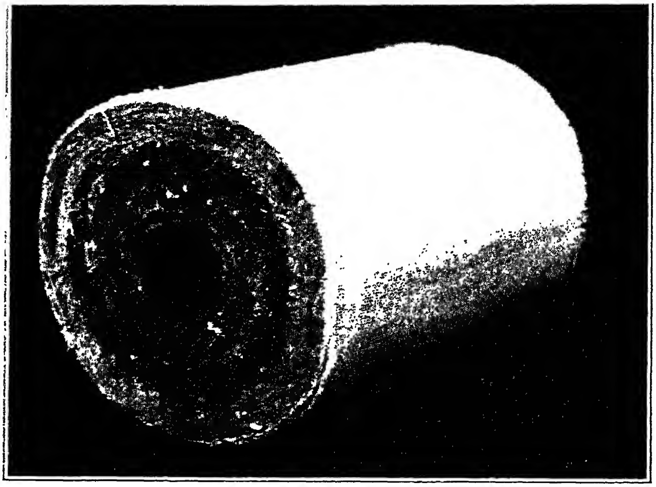


FIG. II–4. Pipe covering used to prevent sweating.

used in many forms such as sheets, blocks, or bulk. Figure II–3 shows a section of cork block. In Fig. II–4 is to be seen a combined low-temperature pipe insulator designed to prevent sweating and to protect against moisture.

(b) Insulation for Building Purposes

This type of insulation is used primarily for reducing the heat loss, from building walls, roofs, etc., during cold weather and the heat penetration during warm weather. Aside from cork, many special insulations as rock wool, slag wool, and glass wool are used. The latter three consist of fine fibers which resemble the general appearance of raw cotton. The fibers are made by blowing the molten material into

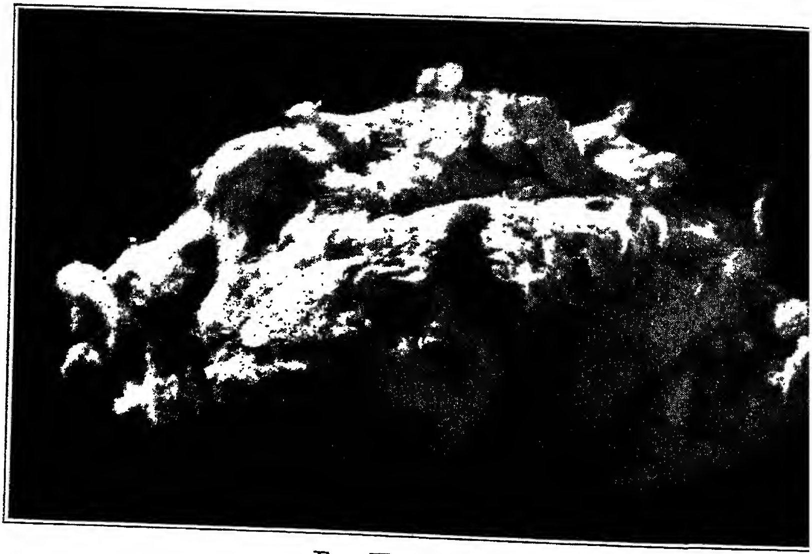


FIG. II–5. Rock wool.

FIG. II–6. Glass wool.

thread form. These insulators are applied in bulk form, blankets, and special reinforced batts. Samples of rock wool and glass wool are shown in Figs. II-5 and 6 respectively.

Metallic or reflecting type insulation is also used in this field. It consists essentially of very thin metal foils which, for instance, can be fastened to building paper. The insulating effect of such metallic surfaces will be explained later. (See Sect. XII-4.)

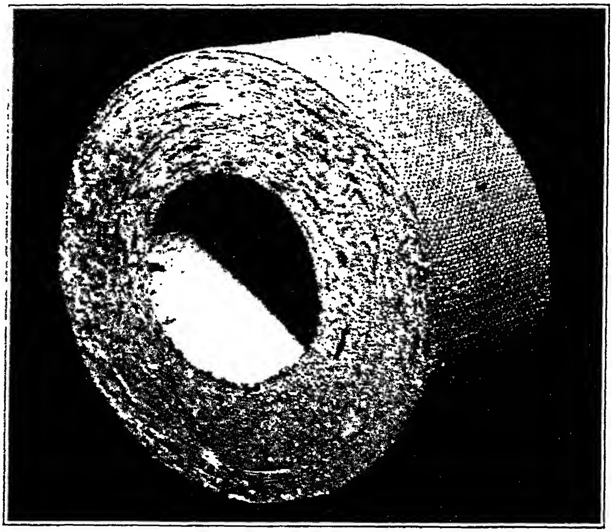


FIG. II-7. Asbestos paper pipe insulation.

(c) *Insulation for Heating and Process Work*

Owing to the relatively low temperatures encountered in this field (150 to 300 F) asbestos paper structures are extensively used. They have the advantages of a rather small conductivity and low cost. Asbestos paper is not suited to much higher temperatures because of the decomposition of the binder. Figure II-7 illustrates a type of pipe insulation made up of layers of asbestos paper having thin air spaces between layers. The structure is to be seen clearly in the figure. Figure II-8 shows a section of corrugated asbestos paper insulation board. The same type is also made in the form of pipe insulation.

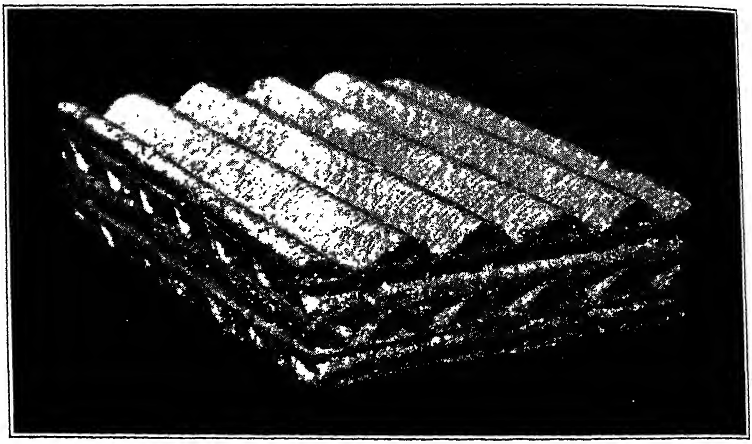


FIG. II-8. Corrugated asbestos paper board insulation.

(d) Insulation in the Power Generation Field

In this field the temperatures encountered are usually between 3 and 600 F. Because the resistance of the insulation to mechani vibration and shock must be an insulation composed of 85% magnesia and 15% asbestos called "85% magnesia" is often used. It is very effective and has high reuse value. Asbestos sponge felt and many other materials are likewise on the market. Figure II-9 shows a piece of "85% magnesia" pipe insulation. Figure II-9*a* gives the thermal conductivity of the material independent of temperature.

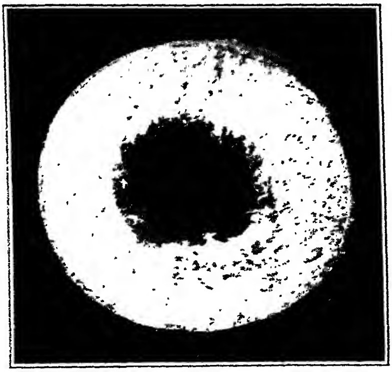


FIG. II-9. Pipe insulation made of 85% magnesia and 15% asbestos.

(e) High-Temperature Insulations

Diatomaceous earth and asbestos are used primarily in this range (600 to 1900 F) because of their resistance to decomposition and high temperatures. Diatomaceous earth is excellent in insulating value, temperature resistance, strength, and durability. It consists of tiny silica skeletons of "diatoms," microscopic plants which have been deposited millions of years ago. Layers of this material have been

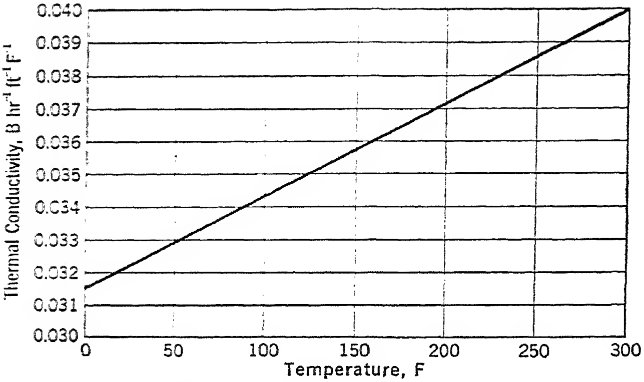


FIG. II-9a. Thermal conductivity of an insulation consisting of 85% magnesia and 15% asbestos having an apparent specific weight of 16.9 lb/ft³.

Data from R. H. Heilman, *Industrial and Engineering Chemistry*, **28**, 782 (1936).

found which are 1400 ft thick. It is used either in natural form (Celite) or in the form of insulating bricks which are made by grinding the earth, pugging, pressing, and firing it in kilns. An insulating brick cut from the material as found in nature is shown in Fig. II-10. Saw marks on the sides and layer marks on the top surface are visible.

Asbestos is a silicate mineral having the characteristic properties of fibrous structure and fire resistance. Diatomaceous earth and asbestos are often mixed together to form effective insulations for this temperature range.

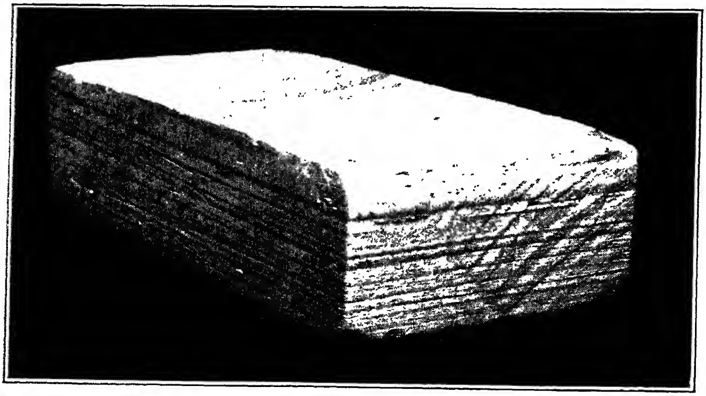


FIG. II-10. Insulating brick cut from natural Celite.

and in sufficient thickness

FIG. II–11. Combined pipe insulation.

than 600 F on the face. A layer of " nesi the -temperature ins tion to decrease further heat loss. Figure II–11 - piece of combined high-temperature and "85% magnesia" insulations for use on high-temperature pipes.

Many other types of insulations are on the market, but a complete discussion is beyond the scope of this text.

In Table II–3 are given data relative to the conductivity of some insulating materials together with the apparent specific weight and

TABLE II–3*

THERMAL CONDUCTIVITY OF SOME INSULATING MATERIALS

Insulation	Apparent Specific Weight lb/cu ft	Thermal Conductivity [B hr^{-1} ft^{-1} F^{-1}] at 20 F	100 F	200 F
Corrugated asbestos (4 plies per inch)	13.0	0.0354	0.0430	0.0525
Glass wool	1.5	0.0217	0.0313	0.0435
	4.0	0.0179	0.0239	0.0317
	6.0	0.0163	0.0218	0.0288
Hair felt	8.2	0.0237	0.0269	0.0310
	11.4	0.0212	0.0254	0.0299
	12.8	0.0233	0.0262	0.0295
% magnesia "	16.9	0.0321	0.0344	0.0373
Rock wool	4.0	0.0150	0.0224	0.0317
	8.0	0.0171	0.0228	0.0299
	12.0	0.0183	0.0226	

* Data taken from R. H. Heilman, *Ind. and Eng. Chemistry*, **28**,

mean temperature. The thermal conductivity of powdered diatomaceous earth for various apparent specific weights can be taken from Table II–4. Examination of the data in these tables indicates that the thermal conductivity of insulating materials increases with temperature and apparent specific weight. The increase due to temperature is greater for the lower apparent specific weights.

TABLE II–4*

THERMAL CONDUCTIVITY OF POWDERED DIATOMACEOUS EARTH

Apparent Specific Weight lb cu ft	Thermal Conductivity [B hr^{-1} ft^{-1} F^{-1}] at 100 F	200 F	300 F	400 F	500 F	600 F
10	0.024	0.029	0.034	0.038	0.044	0.048
14	0.030	0.033	0.036	0.039	0.043	0.046
18	0.038	0.040	0.043	0.045	0.048	0.049

* Data taken from *Diatomaceous Earth* by R. Calvert, Chemical Catalog Co., Inc., 1930.

The size and number of the spaces within the insulating material as well as the arrangement of the fibers in fibrous materials have a marked influence on the thermal conductivity.

II–6 Refractory Materials

In the design and construction of furnaces and related apparatus refractory materials are used owing to their ability to withstand high temperatures without serious deterioration. It is sometimes desirable to select refractory materials of low thermal conductivity, whereas in other cases, good conductors of heat are required. Table II–5 shows the thermal conductivity of twelve different types.

From the data on kaolin bricks in this table it is apparent that for a given temperature the thermal conductivity decreases as the porosity increases. This comes from the insulating effect of small air spaces which has already been mentioned.

The thermal conductivity of most of the refractories quoted in Table II–5 increases with temperature. The opposite is true with the magnesite brick and the silicon carbide brick. The reason for this different behavior must be sought in the inner structure of the refractory materials (Ref. II–1). It has been found, theoretically as well as by experiments, that the thermal conductivity of crystals varies as the reciprocal of the absolute temperature, whereas amorphous (glassy) substances show an increase of the conductivity with the

TABLE II-5*

THERMAL CONDUCTIVITY OF REFRACTORY MATERIALS

Refractory	Apparent Specific Weight lb/ft³	Porosity %	Approximate Chemical Composition %										Thermal Conductivity [B hr⁻¹ ft⁻¹ F⁻¹] at			
			SiO_2	Al_2O_3	Cr_2O_3	SiC	ZrO_2	MgO	CaO	Fe_2O_3	FeO	TiO	392 F	1112 F	1832 F	2552 F
Missouri firebrick	165	18.4	53.1	43.3	...	...	...	0.5	0.6	2.5	...	...	0.58	0.84	0.94	1.00
Pennsylvania firebrick	162	26.7	54.2	38.8	...	...	...	1.1	0.1	2.7	...	2.7	0.58	0.72	0.82	0.89
Kaolin brick	166	10.8	52.0	45.9	...	...	...	...	...	1.5	...	0.4	1.13	1.35	1.54	1.69
Kaolin brick	168	23.2	52.0	45.9	...	...	...	...	...	1.5	...	0.4	0.82	0.99	1.08	1.16
Kaolin brick	156	49.1	52.0	45.9	...	...	...	...	...	1.5	...	0.4	0.27	0.43	0.53	0.58
Silica brick	140	30.4	97.0	...	...	...	...	...	...	...	...	...	0.68	0.96	1.16	1.30
Chrome brick	246	30.5	(5)†	(60)		...	...	(15)	...	...	(15)	...	0.82	0.94	0.96	0.99
Magnesite brick	221	31.6	...	...	...	...	...	(90)	...	(5)	...	...	3.30	2.48	2.17	2.04
Spinel brick	227	36.3	...	65.0	...	...	...	(26)		...	...	...	0.87	1.05	1.13	1.23
Fused alumina brick	229	21.3	...	...	...	...	...	...	...	...	...	...	1.49	1.97	2.29	2.53
Zirconia brick	304	29.5	27.3	7.8	...	...	60.4	...	...	1.6	...	...	0.84	1.05	1.11	1.18
Silicon carbide brick	199	35.3	...	...	...	(100)	...	...	...	...	...	...	...	1.10	0.80	0.63

* Data taken from F. H. Norton, *J. Am. Ceram. Soc.*, **10**, 30 (1927).

† The values in parentheses are round means of values, given in "Modern Refractory Practice" by the Harbison-Walker Refractories Co.

temperature. So in a complex refractory material the crystalline components act in the one direction, the glassy components, the air in the pores, and the radiation through the pores act in the opposite direction as concerns the influence of temperature on the apparent heat conductivity. The two mentioned bricks apparently consisted mainly of crystalline constituents. The great value of the thermal conductivity of the magnesite brick at 392 F is a consequence of this behavior. For the same temperature, heat conductivities of magnesite bricks up to $k > 3$, and for silicon carbide bricks up to $k > 11$ have been observed by reliable investigators.

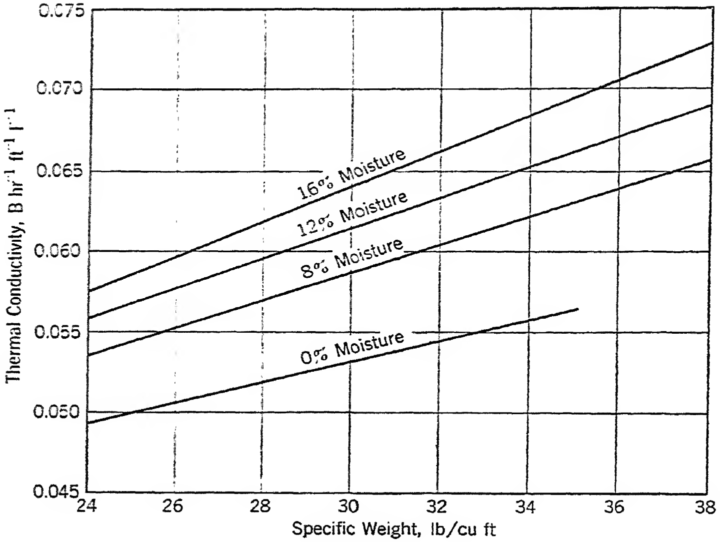


FIG. II–12. Thermal conductivity of Douglas fir at a mean temperature of 75 F.

Data taken from "Thermal Conductivity of Building Materials," by F. B. Rowley and A. B. Algren, *Bulletin* 12, Engineering Experiment Station, University of Minnesota, 1937.

II–7 Building Materials

The thermal conductivity of wood depends upon such factors as moisture content, density, and temperature. Figure II–12 shows the variation of the thermal conductivity of Douglas fir for various specific weights and moisture contents at a mean temperature of 75 F. From the figure it is seen that for a given moisture content the thermal conductivity varies linearly with the specific weight and that it increases

with the moisture content for a given specific weight. Figure II-13 shows the thermal conductivity of various woods of different specific weights at a constant moisture content of 12 per cent. A straight line represents the results fairly well.*

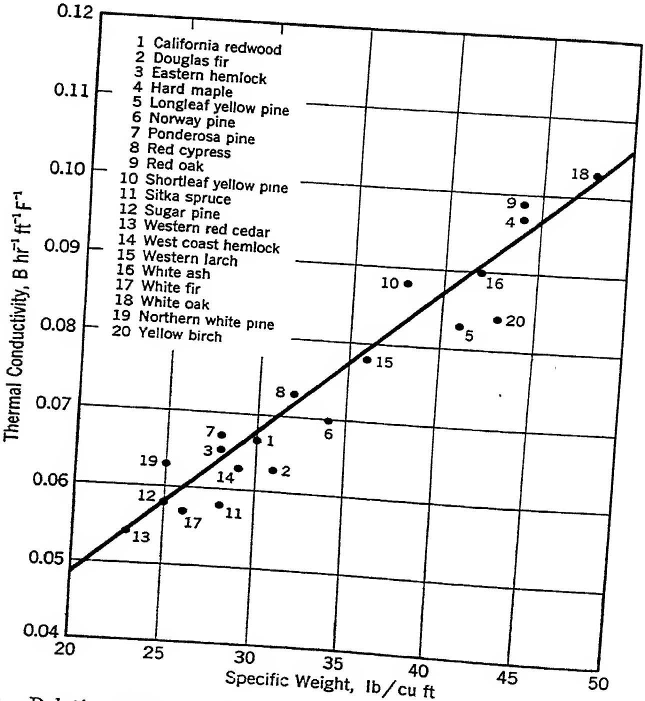


FIG. II–13. Relation between thermal conductivity and average specific weight for twenty different woods at a mean temperature of 75 F and 12 per cent moisture content.

Data taken from "Thermal Conductivity of Building Materials," by F. B. Rowley and A. B. Algren, *Bulletin* 12, Engineering Experiment Station, University of Minnesota, 1937.

Thermal conductivities for various other materials are given in Table II–6. Again the strong influence of the moisture content is striking.

* The most recent results of experiments of J. D. MacLean on various woods [see *Mech. Eng.* **63**, 734 (1941)] came to our knowledge too late for use in preparing this text.

TABLE II-6

THERMAL CONDUCTIVITY FOR VARIOUS BUILDING MATERIALS

Material	Apparent Specific Weight lb/cu ft	Thermal Conductivity B hr^{-1} ft^{-1} F^{-1}
Asphalt	130	0.40
Brick, dry	105	0.30
1% moisture by volume	105	0.40
2% moisture by volume	105	0.60
Concrete, dry	120	0.45
10% moisture by volume	140	0.70
reinforced	140	0.75
Glass	...	0.5 to 0.6
Gypsum, dry	80	0.25
Limestone, fine grain, dry	105	0.40
fine grain, 15% moisture by volume	105	0.55
coarse grain, dry	125	0.55
Linoleum	75	0.11
Plastic, with 2% moisture by volume	115	0.5 to 0.6
Rubber, vulcanized, soft, with 40% pure rubber	...	0.17
with 90% pure rubber	...	0.10
Sand, dry	95	0.20
10% moisture by volume	100	0.60
Soil, dry	125	0.30
fresh clay, 28% moisture by volume	125	1.35

II-8 Metals and Alloys

Pure metals are the best heat conductors. Because of their crystalline structure, the thermal conductivity increases inversely to the absolute temperature as with non-metallic crystals. Impurities, however, disturb the free conduction; they have not only the effect of reducing the heat conductivity appreciably, but also they likewise may change the decrease of the conductivity to an increase with temperature in the higher temperature range. Alloys generally behave like amorphous substances having relatively small thermal conductivities which increase with temperature; but a decrease is sometimes observed as well.

Table II-7 gives values of the thermal conductivity of entirely or rather pure metals. The mentioned influence of temperature and the effect of even small impurities will be noticed.

Values for a few non-iron alloys are given in Table II-8. Here the increase of the conductivity with the temperature is general, and it is

TABLE II–7

THERMAL CONDUCTIVITY OF DIFFERENT METALS

Metal	Impurities %	Thermal Conductivity [B hr^{-1} ft^{-1} F^{-1}] at −58 F	32 F	392 F	752 F	1112 F
Aluminum	0.3	134	132	131	131	. . .
	1.0	116	115	114	. . .	. . .
Copper	0.1	. . .	220	215	210	206
Iron	0.1	. . .	43	36	28	22
Lead	0.0	22	21	19	. . .	. . .
Nickel	0.1	. . .	. . .	42	34	. . .
	1.0	34	35	34	34	34
Platinum	0.05	41	40	42	45	47
Silver	0.0	243	242	. . .	. . .	. . .
	0.1	236	233	217	204	225
Zinc	0.2	67	66	62	54	. . .

TABLE II–8

THERMAL CONDUCTIVITY OF SOME NON-IRON ALLOYS

Material	Thermal Conductivity [B hr^{-1} ft^{-1} F^{-1}] at −58 F	32 F	392 F	752 F
Brass: 70% Cu, 30% Zn	55	60	75	80
"Chromel A": 80% Ni, 20% Cr	..	..	9	11
"Chromel P": 90% Ni, 10% Cr	..	..	12	14
Gunbronze: 86% Cu, 9% Sn, 4% Zn	..	..	38	44
Monel: 67% Ni, 29% Cu	..	..	18	20

further seen that some alloys conduct the heat less than any of their constituents.

Table II–9 contains values for some iron alloys, all of them heat-treated. It is seen that increasing chromium and nickel contents reduce the conductivity to about one-half or one-third the value for wrought iron. The decrease of the thermal conductivity with increasing temperature turns to an increase only for the high-grade chromium and nickel steels.

The range in values of the thermal conductivity for various gases, vapors, liquids, and solids is represented in Fig. II–14.

TABLE II-9

THERMAL CONDUCTIVITY OF SOME HEAT-TREATED IRON ALLOYS*

Material	Chemical Composition %							Thermal Conductivity [B hr^{-1} ft^{-1} F^{-1}] at				
	C	Mn	P	S	Si	Ni	Cr	212 F	392 F	572 F	752 F	932 F
Wrought iron	0.04	0.05	0.136	0.025	0.265	...	...	34.0	31.4	28.7	26.1	23.4
High-carbon steel	0.83	0.27	0.017	0.015	0.16	...	...	26.4	25.1	23.8	22.5	21.2
Low-nickel steel	0.35	0.56	0.015	0.020	0.20	1.37	0.46	25.7	24.7	23.6	22.6	21.5
Low-chromium steel	0.10	0.45	0.013	0.017	0.18	...	5.15	21.1	20.7	20.3	19.8	19.4
Chromium steel	0.07	0.09	0.015	0.010	0.09	0.23	12.00	14.4	14.9	15.5	16.0	...
High-chromium steel	0.10	0.40	0.013	0.008	0.45	0.18	26.00	12.1	12.6	13.2	13.7	14.0
Chromium-nickel steel	0.07	0.27	...	...	...	9.10	18.60	9.5	10.2	11.0	11.7	12.5

* Data taken from "Thermal Conductivity of Some Iron and Steels Over the Range 100 to 500 C," by S. M. Shelton, *Research Paper*, RP 669, U. S. Department of Commerce, Bureau of Standards, April, 1934.

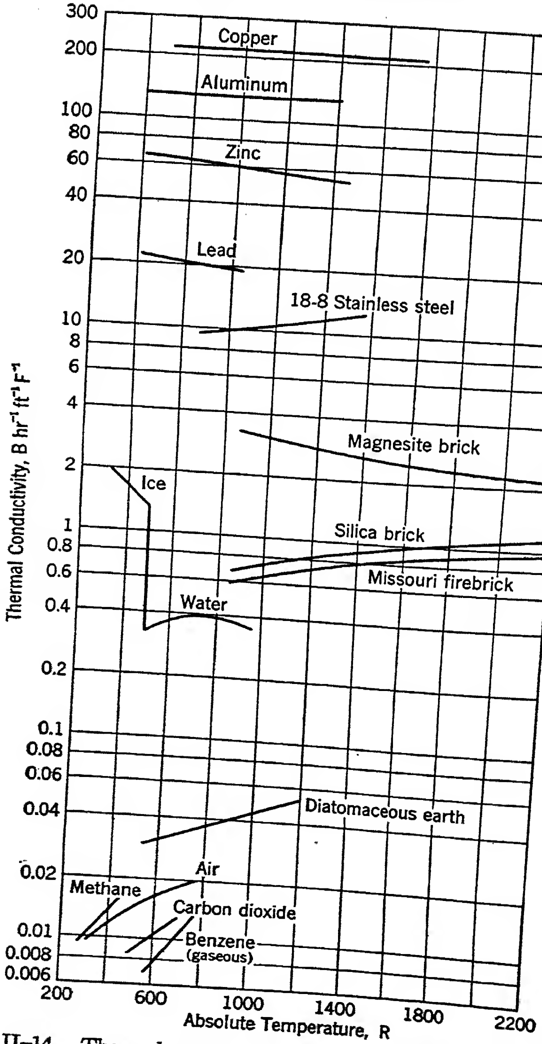


FIG. II–14. Thermal conductivity of gases, liquids, and solids.

REFERENCE

II–1. M. JAKOB, "Structure and Thermal Conductivity of Refractory Bricks,"* *Z. Ver. deutsch. Ing.* **67**, 126 (1923).

* Title translated by the authors.

CHAPTER III

CONDUCTION OF HEAT IN THE STEADY STATE

III–1 Definition of Steady State

A steady state transmission of heat by conduction exists whenever the flow of thermal energy during equal intervals of time is constant, and the temperatures at various positions within the material remain fixed. In order to apply the basic law (Eq. I–1) to different specific cases, it may be expressed in the following differential form:

$$q = -kA\frac{dt}{dx} \qquad \text{[III–1]}$$

The negative sign indicates that the temperature decreases as the distance from a reference point in the direction of the heat flow increases. This formula may also be used if the area and thermal conductivity both vary with x. By integrating the equation, there results a number of equations which can be used for solving problems which occur in engineering practice. In this chapter only cases where the surface temperatures of the bodies under consideration are known will be discussed.

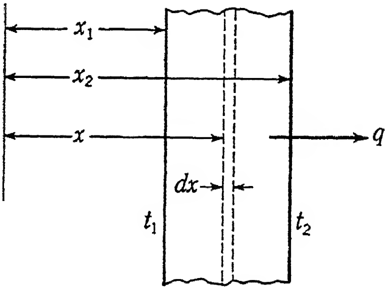


Fig. III–1. Homogeneous plane wall.

III–2 Conduction Through a Homogeneous Plane Wall

It will be assumed that the material composing the wall is homogeneous and that the thermal conductivity of the wall material is independent of the temperature. The direction of the heat flow together with the two surface temperatures is indicated in Fig. III–1. The first step necessary in the analysis of the problem is the separation of the variables in Eq. III–1. This leads to

$$dx \qquad -kA \cdot dt$$

Integrating this relation gives

$$x = \frac{-kAt}{q} + C$$

where C represents a constant of integration which may be det from the following boundary conditions (see Fig. III

When $x = x_1$, $t = t_1$ and when $x = x_2$, $t = t_2$. conditions in the integrated relation gives the foll

$$x_1 = \frac{-kAt_1}{q} + C$$

$$x_2 = \frac{-kAt_2}{q} + C$$

The constant C is eliminated by subtracting the lower equation from the upper:

$$x_1 - x_2 = \frac{-kAt_1}{q} - \frac{-kAt_2}{q}$$

Solving this equation for q gives a relation useful for calculating the heat flowing through the wall:

$$q = kA\,\frac{t_1 - t_2}{x_2 - x_1} \qquad \text{[III-2]}$$

In this equation $(t_1 - t_2)$ and $(x_2 - x_1)$ represent the temperature difference and the thickness of the wall respectively, and A and k refer to the area and thermal conductivity of the material. Introducing $\Delta t = t_1 - t_2$ and $\Delta x = x_2 - x_1$, Eq. III-2 takes the form from which the development started, namely

$$q = kA\,\frac{\Delta t}{\Delta x} \qquad \text{[II-1]}$$

but now without the restriction that Δx and Δt be small.

If one expresses q in B/hr,* A in sq ft, Δt in F,* and Δx in ft, then in order to have consistent units, k must be in B hr^{-1} ft^{-1} F^{-1}. The unit ft^{-1} comes from $A/\Delta x$, which indicates the necessity to convert the thickness Δx into foot units when it is given in inch units.

In practice k is often expressed in

$$\frac{\text{B/sq ft}}{\text{hr} \cdot \text{F/in.}} \text{ units}$$

* See footnote, p. 5.

where the cross-sectional area is expressed in sq ft and the thickness in inch units. This practice is objectionable in that it leads to complications and errors when bodies other than plane plates are to be dealt with.

Example III–1. The interior of an oven is maintained at a temperature of 1500 F by means of suitable control apparatus. If the walls of the oven are 9 in. thick and constructed from a material having a thermal conductivity of 0.18 B hr^{-1} ft^{-1} F^{-1}, calculate the heat loss for each square foot of wall surface per hour. Assume that the inside and outside wall temperatures are 1500 F and 400 F respectively.

Solution: Equation III–2 applies here. Substituting the necessary values and dimensions, particularly $\Delta x = 9/12$ ft, in the equation, gives

$$q = 0.18(1)\,\frac{1500 - 400}{9/12} = 264 \text{ B/hr}$$

or

$$q/A = 264 \text{ B hr}^{-1}\,\text{ft}^{-2}$$

This answer indicates that 264 British thermal units will flow through each square foot of wall area per hour.

III–3 Conduction Through a Composite Plane Wall

Equation III–2 will now be used to establish a relation for the heat flow through a composite wall shown in Fig. III–2. As in the previous case, it will be assumed that the thermal conductivities for the various materials composing the wall are independent of the temperatures. The heat flow through an area A is the same for each section and may be represented as:

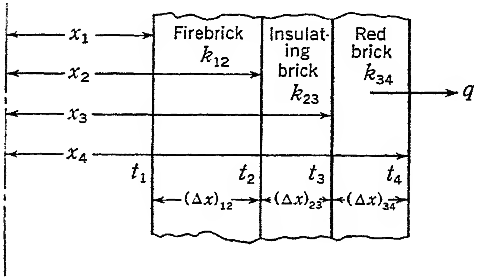


Fig. III–2. Composite plane wall.

$$q = \frac{k_{12}A\,(t_1 - t_2)}{(\Delta x)_{12}} \quad \text{for the firebrick}$$

$$q = \frac{k_{23}A\,(t_2 - t_3)}{(\Delta x)_{23}} \quad \text{for the insulating brick}$$

$$q = \frac{k_{34}A\,(t_3 - t_4)}{(\Delta x)_{34}} \quad \text{for the red brick}$$

Solving for the various temperature differences:

$$t_1 - t_2 = \frac{q(\Delta x)_{12}}{k_{12}A}$$

$$t_2 - t_3 = \frac{q(\Delta x)_{23}}{k_{23}A}$$

$$t_3 - t_4 = \frac{q(\Delta x)_{34}}{k_{34}A}$$

Then by adding these three temperature differences the following is obtained:

$$t_1 - t_4 = \frac{q}{A}\left[\frac{(\Delta x)_{12}}{k_{12}} + \frac{(\Delta x)_{23}}{k_{23}} + \frac{(\Delta x)_{34}}{k_{34}}\right]$$

or

$$q = \frac{A(t_1 - t_4)}{\dfrac{(\Delta x)_{12}}{k_{12}} + \dfrac{(\Delta x)_{23}}{k_{23}} + \dfrac{(\Delta x)_{34}}{k_{34}}} \qquad \text{[III-3]}$$

This equation may be generalized for any number of layers. Calling the total temperature difference $\sum(\Delta t)$ and using the summation sign $\sum$ also in the denominator, gives

$$q = \frac{A\sum(\Delta t)}{\sum \dfrac{\Delta x}{k}}$$

Example III–2. The wall of a furnace is made up of 9 in. of firebrick, 5 in. of insulating brick, and 7.5 in. of red brick. If the average values of the thermal conductivities for the firebrick, insulating brick, and red brick are 0.72, 0.08, and 0.5 B hr^{-1} ft^{-1} F^{-1} respectively, calculate the temperatures t_2 and t_3 at the contact surfaces of the firebrick and insulating brick, and of the insulating brick and red brick respectively. Assume that the inner and outer surface temperatures t_1 and t_4 of the wall are 1500 F and 150 F respectively, and that the resistances of the mortar joints are negligible.

Solution: The rate of heat flow through the wall is obtained from Eq. III–3.

$$q = \frac{1(1500 - 150)}{\dfrac{9/12}{0.72} + \dfrac{5/12}{0.08} + \dfrac{7.5/12}{0.5}} = 180 \text{ B/hr}$$

Since this value of q is equal to the heat flow through any section, Eq. III–2 will be used to calculate the temperature t_2 by applying it to the firebrick section.

$$q = 0.72(12/9)\,(1500 - t_2)$$

which yields $t_2 = 1312$ F. In like manner

$$t_3 = 375 \text{ F}$$

III-4 Heat Resistance

The heat resistance or thermal resistance, R, is defined as the ratio of the length of the heat flow path to the product of the thermal conductivity and the area. Units of the heat resistance are B^{-1} hr F. In terms of this concept, Eq. III-2 may be written in the following manner:

$$q = \frac{t_1 - t_2}{R}$$

This relation is similar to Ohm's law, $I = \dfrac{E}{R_e}$. The analogous terms in the two equations are electrical potential (E) and thermal potential ($t_1 - t_2$), the electrical resistance (R_e) and heat resistance (R), and the electrical current (I) and heat flow (q). Equation III-3 reduces to the following if the resistance term R is used:

$$q = \frac{t_1 - t_4}{R_{12} + R_{23} + R_{34}} \qquad \text{[III-3}a\text{]}$$

Accordingly, the solution of Ex. III-2 on this basis becomes

$$q = \frac{1500 - 150}{1.04 + 5.21 + 1.25} = 180 \text{ B/hr}$$

III-5 Conduction Through a Homogeneous Cylinder Wall

For this case a cylindrical section of insulation surrounding the pipe shown in Fig. III-3 will be used in the analysis. It will again be assumed that the insulating material is homogeneous and that the thermal conductivity of the material is independent of temperature. The rate of the heat which flows radially through the differential volume $2\pi r \cdot dr \cdot L$ is

$$q = -k \cdot 2\pi r \cdot L \frac{dt}{dr} \qquad \text{[III-4]}$$

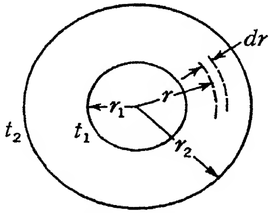


FIG. III-3. Homogeneous cylinder wall.

In this relation r represents the variable radius of the pipe and L the length of the section. For simplification, a linear foot of pipe will be selected so that L equals unity. In order to keep the equation consistent in units, $q' = q/L$ in B hr^{-1} ft^{-1} is introduced. Equation III-4 will now be prepared for integration by

grouping the terms as follows:

$$dt = -\frac{q'}{2\pi k}\frac{dr}{r}$$

Integrating,

$$t = -\frac{q'}{2\pi k}\ln r + C \qquad \text{[III-5]}$$

The constant of integration C will be determined from the following boundary conditions (see Fig. III–3).

When $r = r_1$, $t = t_1$ and when $r = r_2$, $t = t_2$. Substituting this in Eq. III–5 one obtains

$$t_1 = -\frac{q'}{2\pi k}\ln r_1 + C$$

$$t_2 = -\frac{q'}{2\pi k}\ln r_2 + C$$

C will be eliminated by subtracting the lower equation from the upper:

$$t_1 - t_2 = -\frac{q'}{2\pi k}\ln\frac{r_1}{r_2}$$

Solving this relation for q' gives the equation for calculating the heat passing radially through a cylindrical section of pipe insulation 1 ft in length:

$$q' = \frac{2\pi k(t_1 - t_2)}{\ln r_2/r_1} \qquad \text{[III-6]}$$

Example III–3. Calculate the heat loss from 30 ft of 2-in. nominal pipe covered with 1 in. of an insulating material having an average thermal conductivity of 0.0375 B hr^{-1} ft^{-1} F^{-1}. Assume that the inner and outer surface temperatures of the insulation are 380 F and 80 F respectively.

Solution: The outside diameter of a 2-in. nominal pipe is 2.375 in. Adding 2 in. to the pipe diameter, the insulation diameter is equal to 4.375 in. Applying Eq. III–6, the heat loss per linear foot is equal to

$$q' = \frac{2\pi 0.0375(380 - 80)}{\ln\dfrac{4.375/12}{2.375/12}} = 116 \text{ B hr}^{-1}\text{ ft}^{-1}$$

The total heat loss for 30 ft of pipe is equal to (116) (30) = 3480 B/hr.

III–6 Conduction Through a Composite Cylinder Wall

Equation III–6 will be used to establish a relation for calculating the flow of heat per linear foot through the insulation shown in Fig. III–4. The heat flow through each section is the same in the steady state and

is represented by

$$q' = \frac{2\pi k_{12}(t_1 - t_2)}{\ln r_2/r_1} \quad \text{for the inner layer}$$

$$q' = \frac{2\pi k_{23}(t_2 - t_3)}{\ln r_3/r_2} \quad \text{for the outer layer}$$

Solving for the temperature differences

$$t_1 - t_2 = \frac{q' \ln r_2/r_1}{2\pi k_{12}}$$

$$t_2 - t_3 = \frac{q' \ln r_3/r_2}{2\pi k_{23}}$$

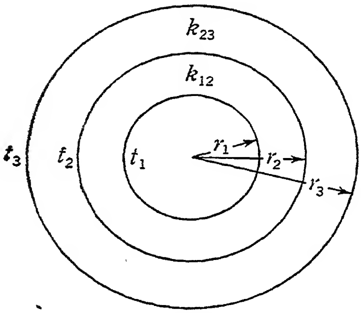


FIG. III-4. Composite cylinder wall.

By adding these two temperature differences

$$t_1 - t_3 = \frac{q'}{2\pi}\left(\frac{\ln r_2/r_1}{k_{12}} + \frac{\ln r_3/r_2}{k_{23}}\right)$$

or

$$q' = \frac{2\pi(t_1 - t_3)}{\left(\dfrac{\ln r_2/r_1}{k_{12}} + \dfrac{\ln r_3/r_2}{k_{23}}\right)} \qquad \text{[III-7]}$$

Generalizing this relation for the case of $(n - 1)$ layers, one obtains

$$q' = \frac{2\pi(t_1 - t_n)}{\sum \dfrac{(\ln r_{m+1}/r_m)}{k}}$$

where $\sum$ indicates a sum to be extended over all integers from $m = 1$ to $m = n - 1$.

Example III–4. Calculate the heat loss per linear foot from a 10-in nominal pipe (outside diameter = 10.75 in.) covered with a composite pipe insulation consisting of $1\frac{1}{2}$ in. of insulation I placed next to the pipe and 2 in. of insulation II placed upon insulation I. Assume that the inner and outer surface temperatures of the composite insulation are 700 F and 110 F respectively, and that the thermal conductivity of material I is 0.05 B hr^{-1} ft^{-1} F^{-1} and for material II is 0.039 B hr^{-1} ft^{-1} F^{-1}.

Solution: Using Eq. III–7 the heat loss is found to be

$$\frac{2\pi\,(700-110)}{\dfrac{\ln\dfrac{13.750/12}{10.750/12}}{0.05}+\dfrac{\ln\dfrac{17.750/12}{13.750/12}}{0.039}} = 323 \text{ B hr}^{-1}\text{ ft}^{-1}$$

III–7 Influence of Variable Conductivity

In all the previous discussion the thermal conductivity of the material was assumed to be independent of the temperature. This assumption holds only approximately as the conductivity of most materials varies with the temperature. For many insulating materials the variation of the thermal conductivity with the temperature may be represented by the linear function:

$$k = k_0(1 + \kappa t) \qquad \text{[III–8]}$$

where k_0 is the conductivity at the temperature $t = 0$, and κ is a constant (see Fig. II–9a). The equation for the flow of heat by conduction through a plane wall will be derived using Eq. III–8 in place of a constant for the thermal conductivity of the material.

According to Eq. III–1 the heat flow through 1 sq ft of wall area is equal to

$$q'' = -k_0(1 + \kappa t)\,\frac{dt}{dx}$$

where $q'' = q/A$. Integrating

$$-\frac{q''x}{k_0} = \left(t + \frac{\kappa}{2}t^2\right) + C$$

By substituting for the boundary conditions $t = t_1$, at $x = x_1$ and $t = t_2$ at $x = x_2$:

$$-\frac{q''x_1}{k_0} = \left(t_1 + \frac{\kappa}{2}t_1^2\right) + C$$

$$-\frac{q''x_2}{k_0} = \left(t_2 + \frac{\kappa}{2}t_2^2\right) + C$$

By subtracting, C is eliminated, and by rearranging the following is obtained:

$$q'' = k_0\left[1 + \frac{\kappa}{2}(t_1 + t_2)\right]\frac{t_1 - t_2}{x_2 - x_1} = k_m \frac{t_1 - t_2}{x_2 - x_1} \qquad \text{[III-9]}$$

where the quantity $k_m = k_0\left[1 + \frac{\kappa}{2}(t_1 + t_2)\right]$ represents a mean value of the thermal conductivity. Therefore, if a linear function exists between the thermal conductivity and the temperature as given by Eq. III-8, the value of k in Eq. III-2 may be replaced by a value of k taken at the mean temperature. It has been shown (see p. 40 of Ref. I-1) that the same holds for Eq. III-6 and also for the case of a sphere (see Problem III-16).

EXAMPLE III-5. Calculate the heat loss through 1 sq ft of a flat slab of 85% magnesia 15% asbestos insulation 2 in. thick if the surface temperatures are 260 F and 180 F respectively.

Solution: For a mean temperature of 220 F, Fig. II-9*a* yields

$$k_m = 0.0377 \text{ B hr}^{-1}\text{ ft}^{-1}\text{ F}^{-1}$$

and by use of Eq. III-9 the heat loss becomes

$$q'' = 0.0377\,\frac{260 - 180}{2/12} = 18.1 \text{ B hr}^{-1}\text{ ft}^{-2}$$

or

$$q = Aq'' = 18.1 \text{ B/hr}$$

PROBLEMS

III-1. If the heat loss per hour through 1 sq ft of a furnace wall 18 in. thick is 520 B, determine the outside surface temperature of the wall material. Assume an inside surface temperature of 1900 F and an average thermal conductivity of 0.61 B hr^{-1} ft^{-1} F^{-1}.

III-2. The wall of a cold-storage room is made up of 12 in. of cork having a mean thermal conductivity of 0.028 B hr^{-1} ft^{-1} F^{-1}. If the inside and outside surface temperatures are 20 F and 70 F respectively, calculate the temperature at a plane 8 in. from the cold inner surface.

III-3. Calculate the heat loss from a composite furnace wall made up of 9 in. of firebrick, 6 in. of insulating brick, and 4 in. of red brick. The inside and outside surface temperatures of the wall are 2000 F and 110 F respectively. The mean thermal conductivity values for the firebrick, insulating brick, and red brick are 0.7, 0.08, and 1.0 B hr^{-1} ft^{-1} F^{-1} respectively.

III-4. Verify the following: In order to convert k in B hr^{-1} ft^{-1} F^{-1} to watt cm^{-1} C^{-1} multiply by 0.0173.

III-5. Convert a value for k of 60 $\dfrac{\text{B/sq ft}}{\text{hr F/in.}}$ into B hr^{-1} ft^{-1} F^{-1}.

III–6. Calculate the heat passing through a furnace wall 9 in. thick if the inside and outside surface temperatures are 1800 F and 390 F respectively. Assume that the mean thermal conductivity for the wall material is 0.667 B hr^{-1} ft^{-1} F^{-1}.

III–7. If 0.3 in. of insulation ($k = 0.046$ B hr^{-1} ft^{-1} F^{-1}) is added to the outside surface of the wall in Problem III–6 and reduces the heat loss 20 per cent, calculate the outside surface temperature of the wall.

III–8. If the cost of insulation in Problem III–7 is \$1.37 per sq ft, calculate the time required to pay for the insulation. Base the calculation on twenty-four hours' operation per day and one hundred and seventy-five days per year. Heat to be valued at 23 cents per million B.

III–9. A wall of 0.8-ft thickness is to be constructed from material which has an average thermal conductivity of 0.75 B hr^{-1} ft^{-1} F^{-1}. The wall is to be insulated with material having an average thermal conductivity of 0.2 B hr^{-1} ft^{-1} F^{-1} so that the heat loss per square foot will not exceed 580 B per hour. If the inner and outer surface temperatures of the insulated wall are to be 2400 F and 80 F respectively, calculate the thickness of insulation required.

III–10. Prove that the heat loss per square foot of outer surface of an insulated pipe is equal to the following relations:

$$q'' = \frac{2\,k(t_1 - t_2)}{D_2 \ln \dfrac{D_2}{D_1}}$$

$$q'' = \frac{0.868\,k(t_1 - t_2)}{D_2 \log \dfrac{D_2}{D_1}}$$

In these relations D refers to the diameter.

III–11. Prove that the following equation expresses the heat loss per square foot of outside surface area from a pipe insulated with three layers.

$$q'' = \frac{0.868(t_1 - t_4)}{D_4\left(\dfrac{1}{k_{12}} \log \dfrac{D_2}{D_1} + \dfrac{1}{k_{23}} \log \dfrac{D_3}{D_2} + \dfrac{1}{k_{34}} \log \dfrac{D_4}{D_3}\right)}$$

III–12. Calculate the heat loss per square foot of outer surface for a 4-in. nominal pipe covered with ½ in. of insulation ($k = 0.047$ B hr^{-1} ft^{-1} F^{-1}) if the outside and inside temperatures of the insulation are 90 F and 400 F respectively.

III–13. A 6-in. steam main is covered with 2 in. of high-temperature insulation ($k_m = 0.053$ B hr^{-1} ft^{-1} F^{-1}) and 1½ in. of lower-temperature insulation ($k_m =$ 0.041 B hr^{-1} ft^{-1} F^{-1}). Calculate the heat loss from 400 ft. of pipe if the inner and outer surface temperatures of the insulation are 800 F and 100 F respectively.

III–14. Calculate the heat loss per linear foot for an 8-in. steam pipe covered with 1½ in. of high-temperature insulation ($k_m = 0.052$ B hr^{-1} ft^{-1} F^{-1}) and 2½ in. of a lower-temperature insulation ($k_m = 0.041$ B hr^{-1} ft^{-1} F^{-1}). Assume that the inner and outer surface temperatures of the insulation are 800 F and 70 F respectively.

III–15. A 6-in. pipe is covered with 1 in. of insulation I ($k_m = 0.05$ B hr^{-1} ft^{-1} F^{-1}) and 2 in. of insulation II ($k_m = 0.041$ B hr^{-1} ft^{-1} F^{-1}). If the inner and outer surface temperatures of the insulation are 650 F and 100 F respectively, calculate the heat loss for 800 sq ft of outer pipe surface.

III–16. Prove that the heat loss per square foot of outside surface area of a hollow sphere heated from within is equal to

$$,, \quad 2\,k(t_1 - t_2)$$

where t_1 and t_2 are the temperatures and D_1 and D_2 are the diameters of the inner and outer surface respectively.

III–17. Calculate the heat loss per square foot of outside surface area for a heated sphere 6 in. in diameter covered with 2 in. of insulation (k_m = 0.04 B $\text{hr}^{-1}\,\text{ft}^{-1}\,\text{F}^{-1}$). The inside and outside surface temperatures of the insulation are 600 F and 180 F respectively.

III–18. A hollow sphere is heated by means of a heating coil having a resistance of 100 ohms placed in the inside cavity. If the average thermal conductivity of the sphere material is 30 B $\text{hr}^{-1}\,\text{ft}^{-1}\,\text{F}^{-1}$, calculate the current necessary to maintain a temperature difference between the inside and outside surfaces of 8 F. The inside and outside diameters of the sphere are 8 and 9 in. respectively.

III–19. Plot a curve based on calculated data which will indicate the heat loss per linear foot per hour for various thicknesses of insulation on a 1-in. nominal pipe. Assume that the inside and outside surface temperatures of the insulation are 500 F and 100 F respectively. Assume also that the mean value of k is 0.3 B $\text{hr}^{-1}\,\text{ft}^{-1}\,\text{F}^{-1}$.

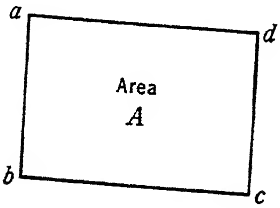
III–20. A hollow sphere whose inner and outer diameters are 14 cm and 16 cm respectively is heated by means of a resistance coil (20 ohm) placed inside the sphere. If k for the sphere material is 20 B $\text{hr}^{-1}\,\text{ft}^{-1}\,\text{F}^{-1}$, compute the current necessary to keep the two surfaces at a steady temperature difference of 4 C and calculate the heat supplied in calories per second.

CHAPTER IV

CONDUCTION OF HEAT IN THE UNSTEADY STATE

IV–1 General Equations for Unsteady State

If the rate of heat flow and the temperature at any point of a system change with time, the condition is termed "unsteady state" or "transient state." Whereas until now only heat conduction in the steady state has been considered, this chapter deals with some of the most important cases of heat conduction under unsteady state conditions. Many industrial problems of heat transmission come under this class of heat flow. One familiar example is the flow of heat through a building wall during the daily twenty-four-hour heating and cooling cycle. Other examples include the annealing of castings, cooling of ingots, burning of bricks, and the heating and cooling of slabs and furnace wall sections.



l. Front view of plate.

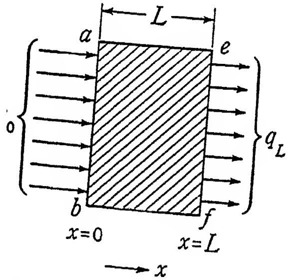


FIG. IV–2. Cross section of plate with incoming and outgoing heat flow.

The general equation for the flow of heat by conduction in the unsteady state will be derived for the simplest case, the warming up of a plane plate which is heated from one side and cooled from the other.

Figures IV–1 and 2 show a front view and a cross section of the plate. Its area may be A and its thickness L. It is assumed that the heat flow be uniformly distributed over the area A and that the edges of the plate, given by $abcd$ in the front view and by ae and bf in the sectional view, are perfectly insulated against heat losses or gains. Then heat will flow only in the direction x.

The rate of flow of the heat which enters at the left surface may be called q_0, that which leaves at the right q_L, both being unvariable with time, and $q_0 > q_L$.

At a certain instant of time τ, the distribution of temperature t across the plate may be given by the curve a in Fig. IV–3a. The rate of heat flow q_x at any distance x from the left-hand surface can be found from this curve by means of Eq. III–1 which in the present case will be written as

$$q_x = -kA \frac{\partial t}{\partial x} \qquad \text{[IV–1]}$$

Here the partial derivative $\partial t/\partial x$ has been chosen instead of dt/dx in order to indicate that the temperature t does not only vary with the distance x, but likewise also with the time τ. The latter change will be indicated by $\partial t/\partial \tau$.

The slope of curve a is $\partial t/\partial x = \tan \phi$. It is often called the "temperature gradient." If t decreases when x increases, $\partial t/\partial x$ is negative. Then, according to Eq. IV–1, q_x becomes positive. This is the case of heat conduction in the direction of increasing x.

For $x = 0$ and $x = L$ the slope $\partial t/\partial x$ can be calculated by substituting the given constant values q_0 and q_L for q_x in Eq. IV–1. Using the subscripts 0 and L for the gradient at $x = 0$ and $x = L$, this equation yields

$$\left(\frac{\partial t}{\partial x}\right)_0 = -\frac{q_0}{kA} \qquad \text{[IV–2]}$$

and

$$\left(\frac{\partial t}{\partial x}\right)_L = -\frac{q_L}{kA} \qquad \text{[IV–3]}$$

Since it was assumed that $q_0 > q_L$, it follows that curve a will be steeper on the left than on the right surface, as shown in Fig. IV–3a.

By taking $\tan \phi$ from this figure for different distances x one finds the distribution of the temperature gradient $\partial t/\partial x$ across the plate. It is represented by the curve b in Fig. IV–3b. Since t decreases from left to right, the ordinates of curve b are negative throughout. According to Eq. IV–1 each ordinate of curve b must be multiplied by $(-kA)$ in order to obtain the heat flow q_x as a function of x. Obviously, all values of q_x become positive in the present case and because curve b is rising from left to right, the heat flow q_x will decrease from left to right. (Curve c in Fig. IV–3c.)

The heat flow, however, according to the law of conservation of energy can decrease only when heat is to be stored in the wall. To find

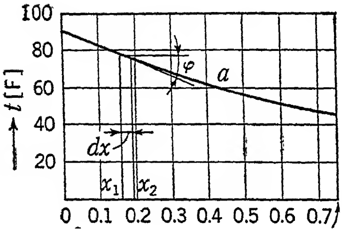


Fig. IV–3a. Temperature distribution (Ex. IV–1).

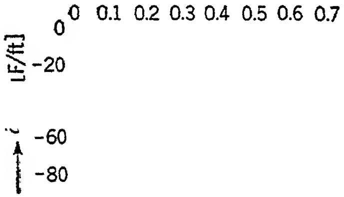


Fig. IV–3b. Temperature gradient (Ex

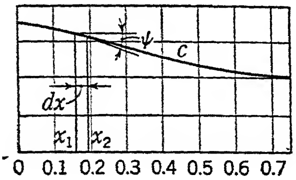


Fig. IV–3c. Heat flow (Ex. IV–1).

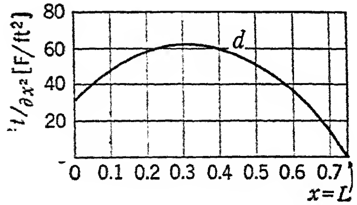


Fig. IV–3d. Slope of temperature gradient (Ex. IV–1).

this storage, a piece of the wall of differential length dx may be considered.

From Fig. IV–3c it is seen that

$$q_2 = q_1 + (\tan \psi_1) dx = q_1 + \left(\frac{\partial q_x}{\partial x}\right)_1 dx \qquad \text{[IV–4]}$$

where the subscript 1 relates to the place x_1 and 2 to the place $x_2 = x_1 + dx$.

From Eq. IV–1 it follows by differentiation that

$$\frac{\partial q_x}{\partial x} = -kA \frac{\partial^2 t}{\partial x^2}$$

When this is substituted in Eq. IV–4 and the subscript of the derivative is omitted

$$q_2 = q_1 - kA \frac{\partial^2 t}{\partial x^2} dx \qquad \text{[IV–5]}$$

or

$$dq_3 = q_1 - q_2 = kA \frac{\partial^2 t}{\partial x^2} dx \qquad \text{[IV–6]}$$

Because q_1 is the rate of heat flow entering the differential piece at x_1 and q_2 that leaving at x_2, their difference, which is called dq_3 in Eq. IV–6, is just the heat energy stored in unit time in a slice of the plate of thickness dx.

As t, $\partial t/\partial x$, and q_x, likewise dq_3 can be represented graphically. It is only necessary to determine graphically in Fig. IV–3b the slope

$$\tan \qquad \frac{\partial(\partial t/\partial x)}{\partial x} = \frac{\partial^2 t}{\partial x^2}$$

for different values of x and then use these values of $\tan \chi$ as ordinates of a curve d in Fig. IV–3d. If the plate is considered as cut in slices of equal thickness dx, the factor $(kA \cdot dx)$ in Eq. IV–6 is constant. It follows that the rate of heat energy stored in the slice dx is found from curve d simply by multiplying each ordinate by the constant factor $(kA \cdot dx)$.

The rate of heat storage dq_3 can also be found in a different way. Multiplying the volume $(A \cdot dx)$ by the density ρ gives the mass. Multiplying the mass $(A \cdot dx \cdot \rho)$ by the specific heat c_p gives the heat stored if the temperature increases by one degree; so if it increases by ∂t in the time $\partial \tau$, the heat stored is

$$dq_3 = A \cdot dx \cdot \rho c_p \frac{\partial t}{\partial \tau} \qquad \text{[IV–7]}$$

Equating dq_3, as given by Eqs. IV–6 and 7, leads to

$$dq_3 = A \cdot dx \cdot \rho c_p \frac{\partial t}{\partial \tau} = kA \frac{\partial^2 t}{\partial x^2} dx \qquad \text{[IV-8]}$$

or

$$\boxed{\frac{\partial t}{\partial \tau} = \frac{k}{\rho c_p} \cdot \frac{\partial^2 t}{\partial x^2} = \alpha \frac{\partial^2 t}{\partial x^2}} \qquad \text{[IV-9]}$$

where

$$\alpha = \frac{k}{\rho c_p} \qquad \text{[IV-9a]}$$

is a property of the wall material which is called "thermal diffusivity."

By contemplating curve d in the light of Eq. IV–9, it is seen that by multiplying the ordinate $\partial^2 t/\partial x^2$ by the magnitude α the change of temperature in time $\partial t/\partial \tau$ is given directly at any distance x and at the time τ. In the present case $\partial t/\partial \tau$ is positive. This means that the temperature all over the cross section of the plate increases with time, which is obvious because it was assumed that more heat enters than leaves the plate.

The physical dimensions of the thermal diffusivity α can be derived from

$$[k] = [\text{B hr}^{-1}\,\text{ft}^{-1}\,\text{F}^{-1}]$$

$$[\rho] = [\text{lb}_m\,\text{ft}^{-3}]$$

$$[c_p] = [\text{B lb}_m^{-1}\,\text{F}^{-1}]$$

Therefore

$$[\alpha] = \left[\frac{k}{\rho c_p}\right] = [\text{ft}^2\,\text{hr}^{-1}]$$

The remarkable feature of this relation is that neither a heat unit nor a temperature unit appears.

Further it is seen that the product $[\rho c_p]$ does not include mass or force units, but is expressed by $[\text{B ft}^{-3}\,\text{F}^{-1}]$. This, of course, holds also if ρ is expressed in [slug/cu ft] and c_p in $[\text{B slug}^{-1}\,\text{F}^{-1}]$. With these units the numerical value of ρ would be only 1/32.2 of the usual value of density, as mentioned in Sect. I–3, and the numerical value of c_p would be 32.2 times as large as the usual value. For water, for instance, $\rho = 1.937$ slug/cu ft has been found (see Sect. I–3), and the specific heat of water is 32.2 B $\text{slug}^{-1}\,\text{F}^{-1}$ instead of the familiar value 1. To bring them back to the usual values one can introduce the specific weight γ instead of the density ρ. Then, according to Eq. I–5, $\gamma = \rho g = (1.937)(32.2) = 62.3\ \text{lb}_f/\text{cu ft}$ and $\rho c_p = \gamma c_p/g$. This means

that using γ instead of ρ requires the use of c_p/g instead of c_p in the product ρc_p. For water the numerical value of $c_p/g = 1$, and the physical dimension of this magnitude becomes $\dfrac{[\mathrm{B\ slug^{-1}\ F^{-1}}]}{[\mathrm{ft\ sec^{-2}}]} = [\mathrm{B\ lb_f^{-1}\ F^{-1}}]$. It is seen that in the pound force system the latter expression is not the unit of the specific heat as often is believed and claimed, but the unit of c_p/g. Thus, in this system Eq. IV–9*a* becomes

$$\frac{k}{\gamma(c_p/g)} \qquad [\text{IV–9}b]$$

Dealing with the product ρc_p only, and not with its single factors, the whole difficulty can be avoided by using a special symbol $C_p = \rho c_p = \gamma(c_p/g)$ with the dimension $[\mathrm{B\ ft^{-3}\ F^{-1}}]$, that is a specific heat at constant pressure related to unit volume instead of unit mass. Then Eqs. IV–9*a* and *b* assume the common form

$$\alpha = \frac{k}{C} \qquad [\text{IV–9}c]$$

Example IV–1. The distribution of the temperature t across a large plane dry concrete wall 9 in. thick, which is heated from one side, is measured by thermocouples inserted in holes in the wall, and it has been found that at a certain instant τ, the temperature can be represented approximately by the equation

$$t = 90 - 80x + 16x^2 + 32x^3 - 25.6x^4 \qquad [\text{IV–10}]$$

where lengths are in feet and temperatures in F.

If the area of the wall is 5 sq ft, calculate (*a*) the heat entering and leaving in unit time, (*b*) the heat energy stored in the wall in unit time, and (*c*) the temperature changes per unit of time.

Solution: The specific weight γ of concrete is assumed to be 136 lb/ft^3, the specific heat $c_p = 0.202\ \mathrm{B\ lb_m^{-1}\ F^{-1}} = 0.202(32.2)\ \mathrm{B\ slug^{-1}\ F^{-1}}$ and the thermal conductivity $k = 0.44\ \mathrm{B\ hr^{-1}\ ft^{-1}\ F^{-1}}$. Then $C_p = (136/32.2)(0.202)(32.2) = (136)(0.202)$, and the diffusivity $\alpha = k/C_p = 0.016\ \mathrm{ft^2\ hr^{-1}}$. The thickness of the plate must be expressed in feet,

$$L = \tfrac{9}{12}\ \text{ft} = 0.75\ \text{ft}$$

By substituting different numerical values for x (from $x = 0$ to $x = L = 0.75$) it will be seen that the temperature distribution is that represented in Fig. IV–3*a*. Also Figs. IV–3*b* to *d* are drawn to scale for the present example.

Differentiation of Eq. IV–10 leads to

$$\frac{\partial t}{\partial x} = -80 + 32x + 96x^2 - 102.4x^3 \qquad [\text{IV–11}]$$

and

$$\frac{\partial^2 t}{\partial x^2} = 32 + 192x - 307.2x^2 \qquad \text{[IV-1}$$

From Fig. IV–3*d* it is seen that $\partial^2 t/\partial x^2$ has a maximum value. Accordin to the rules of calculus the distance x at which it occurs is found by formin the derivative of the last equation and equating it to zero. Thus 192 - $614.4\,x = 0$ and $x = 0.312$ ft. At this point $\partial^2 t/\partial x^2 = 32 + 192(0.312) - 307.2(0.312^2) = 62.1$ F/ft².

(*a*) According to Eqs. IV–2 and 3 the values of $(\partial t/\partial x)_0$ and $(\partial t/\partial x)_L$ ar needed for the calculation of q_0 and q_L. Substituting $x = 0$ and $x = L =$ 0.75 in Eq. IV–11

$$\left(\frac{\partial t}{\partial x}\right)_0 = -80 \text{ F/ft}$$

$$\left(\frac{\partial t}{\partial x}\right)_L = -80 + 24 + 54 - 43.2 = -45.2 \text{ F/ft}$$

and

$$q_0 = -kA\left(\frac{\partial t}{\partial x}\right)_0 = -(0.44)5(-80) = 176 \text{ B/hr}$$

$$q_L = -kA\left(\frac{\partial t}{\partial x}\right)_L = (0.44)5(45.2) = 99.44 \text{ B/hr}$$

(*b*) The heat stored is

$$q_3 = q_0 - q_L = 176 - 99.44 = 76.56 \text{ B/hr}$$

It can also be found from Eqs. IV–8 and 12 by integration:

$$q_3 = \int_{x=0}^{x=L} dq_3 = kA\int_{x=0}^{x=L} \frac{\partial^2 t}{\partial x^2}\,dx = kA\int_{x=0}^{x=L} (32 + 192\,x - 307.2\,x^2)\,dx$$
$$= kA(32\,L + 96\,L^2 - 102.4\,L^3) = (0.44)5(24 + 54 - 43.2)$$
$$= 76.56 \text{ B/hr}$$

(*c*) From Eqs. IV–9 and 12 the following is obtained:

$$\frac{\partial t}{\partial \tau} = \alpha(32 + 192x - 307.2x^2) = 0.016(32 + 192x - 307.2x^2)$$

his means that the ordinates of curve *d* (Fig. IV–3*d*) must be multiplied y $\alpha = 0.016$ in order to get $\partial t/\partial \tau$. Thus at

$x = 0$	$\partial^2 t/\partial x^2 = 32$ F/ft²	$\partial t/\partial \tau = 0.51$ F/hr
$x = 0.312$	$= 62.1$ F/ft²	0.99 F/hr
$x = 0.75$	$= 3.2$ F/ft²	0.05 F/hr

ne temperature of the wall does not change by the same amount at each int in the wall, but by about 0.5 F/hr at the heated surface, 1.0 F/hr at a pth of about 3¾ in. and 0.05 F/hr at the cooled surface.

EXAMPLE IV-2. Answer the same questions as in Ex. IV-1 for a temperature distribution, given by the equation

$$t = 90 - 8x - 80x^2 \qquad \text{[IV-13]}$$

Solution:

$$\frac{\partial t}{\partial x} = -8 - 160x \qquad \text{[IV-13}a\text{]}$$

$$\frac{\partial^2 t}{\partial x^2} = -160 \qquad \text{[IV-13}b\text{]}$$

(*a*) According to Eqs. IV-2 and 3:

$$q_0 = -kA\left(\frac{\partial t}{\partial x}\right)_0 = -(0.44)5(-8) = 17.6 \text{ B/hr}$$

$$q_L = -kA\left(\frac{\partial t}{\partial x}\right)_L = -(0.44)5(-128) = 281.6 \text{ B/hr}$$

(*b*)

$$q_s = q_0 - q_L = -264 \text{ B/hr}$$

The negative sign shows that no heat is stored, but more heat energy is given up at $x = L$ than received at $x = 0$.

This can also be found from Eqs. IV-8 and 13*b*.

$$q_s = \int_{x=0}^{x} dq_s = -0.44(5)160 \int_{x=0}^{x=L} dx = -0.44(5)160(0.75)$$
$$= -264 \text{ B/hr}$$

(*c*) From Eqs. IV-9 and 13*b* the following is obtained:

$$\frac{\partial t}{\partial \tau} = 0.016(-160) = -2.56 \text{ F/hr}$$

Here the temperature of each part of the wall decreases equally.

As in Figs. IV-3*a*, *b*, and *d*, the distribution of t, $\partial t/\partial x$, and $\partial^2 t/\partial x^2$ is represented in Figs. IV-4*a* and *b*.

It is easily understood that the difference in these examples is due to differences in the curvature of curve *a* in Figs. IV-3*a* and 4*a*. In the former the curve is concave, seen from above, and in the latter it is convex. Therefore, the temperature gradient at the left is greater than at the right in the first, and smaller in the second case. This indicates that in Ex. IV-1 more heat is supplied than given up, in Ex. IV-2 the opposite is true, and by this $\partial t/\partial \tau$ is positive in the former, negative in the latter. That the curve *d* simplifies to a horizontal line in Ex. IV-2 comes from the simple form of Eq. IV-13.

Finally it must be kept in mind that $\partial t/\partial \tau$ as found in Ex. IV-1 holds only for the instant τ. At a later instant the temperature distribution

across the plate will be changed. Assuming, however, that $\partial t/\partial \tau$ is practically constant over a reasonable time interval, for instance 1 hr in Ex. IV–1, it is possible to find a temperature curve like curve *a* in Fig. IV–3*a* for a later instant. After 1 hr, according to this calculation, *t* will have increased by 0.5 F at $x = 0$; 1 F at $x = 0.312$; 0.05 F at $x = 0.05$. Using the new curve, which is higher, and repeating the construction given by Figs. IV–3*a* to *d*, the temperature distribution for any later instant may be found.

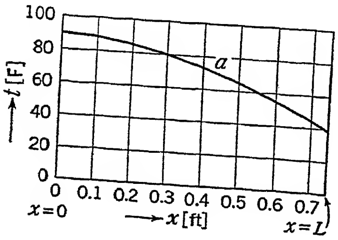


FIG. IV–4*a*. Temperature bution (Ex. IV–2).

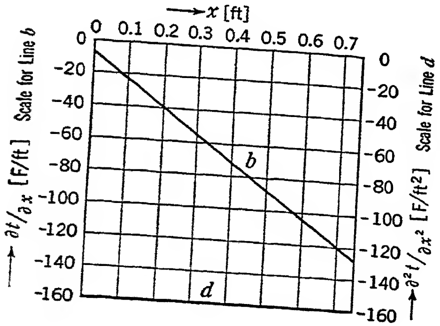


FIG. IV–4*b*. Temperature gradient and its slope (Ex. IV–2)

Because Ex. IV–2 led
rve *a* in Fig. IV–4*a* ... down in one hour by -2.56 F as ... of ole, and the curves *b* and *d* in Fig. IV–4*b* obviously will remain unchanged. Thus, in this case the temperature of any point in the wall would decrease continuously by the same amount. This, however, as will easily be understood, is only possible if the amounts of heat supplied in time unit at the left and rejected at the right surface remain unchanged.

Equation IV–9 has been derived for heat flow only in the direction *x*. For heat flow in any direction in a homogeneous body, and temperature hanges in the directions of the three ... *z*, a similar

derivation leads to the more general equation:

$$\frac{\partial t}{\partial \tau} = \alpha\left(\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}\right)$$

IV–2 Steady State as a Special Case of the General Equation

Since Eq. IV–9 is entirely general for conductive heat flow in one direction if the thermal diffusivity α is invariable, the equation must hold also for steady state conditions.

In this case the temperature at any point is constant with respect to time, i.e., $\partial t/\partial \tau = 0$. Therefore, total differentials can be used again and Eq. IV–9 becomes

$$\frac{d^2t}{dx^2} = 0 \qquad \text{[IV–15]}$$

Integrating once gives

$$dt$$

C' being the constant of integration. A second integration gives

$$t = C'x + C'' \qquad \text{[IV–16]}$$

C'' being a second constant of integration.

The boundary conditions are as follows (see Fig. III–1): When $x = x_1$, $t = t_1$, and when $x = x_2$, $t = t_2$.

Substituting the boundary values in Eq. IV–16 gives

$$t_1 = C'x_1 + C''$$

$$t_2 = C'x_2 + C''$$

Solving the two equations for C' and C''

$$C' = \frac{t_1 - t_2}{\quad - x_2} \quad \text{and} \quad C'' = t_1 -$$

Substituting these relations in Eq. IV–16 gives the following relation for determining the temperature at any position x within the wall, which is called the temperature field equation:

$$t = \frac{\quad - x(t_1 - t_2)}{(x_2 - \quad} \qquad \text{[IV–17]}$$

The solution of Eq. IV–9 is, in general, not so easy as in the very simple case of steady flow conditions just considered. The integration is beyond the level of this text on the elements of heat transfer. Therefore, in the following, only a few of the solutions will be given, but no derivations.

IV–3 Sudden Change of the Surface Temperature of a Thick Plate Wall

First an infinitely thick wall may be considered which is originally at temperature t_i throughout. The surface temperature changes suddenly to t_s and remains constant thereafter. Here the integration of Eq. IV–9 leads to

$$t = t_s + (t_i - t_s)f\left(\frac{x}{\quad}\right.$$

t = the temperature in the wall at a distance x from the surface, and $f\left(\frac{x}{2\sqrt{\alpha\tau}}\right)$ is a function which is called Gauss's error
the time τ after the sudden temperature change

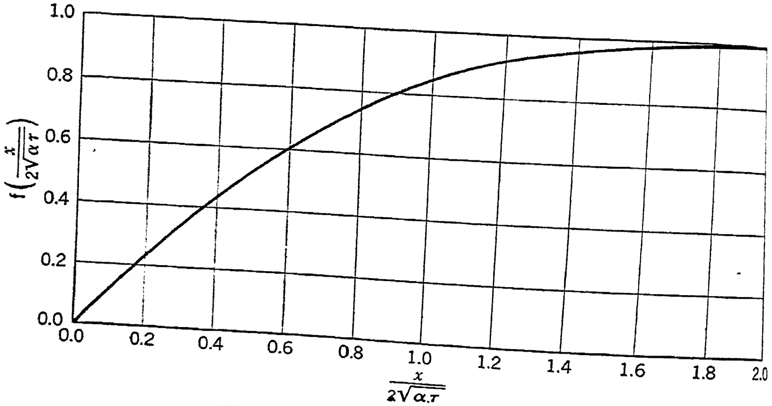


FIG. IV–5. Gauss's error integral.

integral. Numerical values of this funct

plotted in Fig. IV–5. It is seen that is a ex-

if x, τ, and α are in , for instance x in ft.
τ in hr, and α in ft^2/hr.

The rate of heat flow through an area direction
surface into the infinitely thick wall at a nce x and at the time τ
may be found from the equation

[IV–19]

Values of e^{-x} are plotted in Fig.

TABLE IV-1

VALUES OF GAUSS'S ERROR INTEGRAL

$\frac{x}{2\sqrt{\alpha\tau}}$	$f\left(\frac{x}{2\sqrt{\alpha\tau}}\right)$	$\frac{x}{2\sqrt{\alpha\tau}}$	$f\left(\frac{x}{2\sqrt{\alpha\tau}}\right)$	$\frac{x}{2\sqrt{\alpha\tau}}$	$f\left(\frac{x}{2\sqrt{\alpha\tau}}\right)$
0.00	0.0000	0.4	0.4284	1.3	0.9340
0.01	0.0113	0.5	0.5205	1.4	0.9523
0.02	0.0226	0.6	0.6039	1.5	0.9661
0.04	0.0451	0.7	0.6778	1.6	0.9763
0.06	0.0676	0.8	0.7421	1.8	0.9891
0.08	0.0901	0.9	0.7969	2.0	0.9953
0.10	0.1125	1.0	0.8427	2.2	0.9981
0.20	0.2227	1.1	0.8802	2.5	0.9996
0.30	0.3286	1.2	0.9103	3.0	1.0000

Taking $x = 0$ in Eq. IV-19 one finds the heat flow into the surface of the wall at the time instant τ:

$$q = -kA(t_i - t_s) \cdot \frac{1}{\sqrt{\pi\alpha\tau}} \qquad \text{[IV-20]}$$

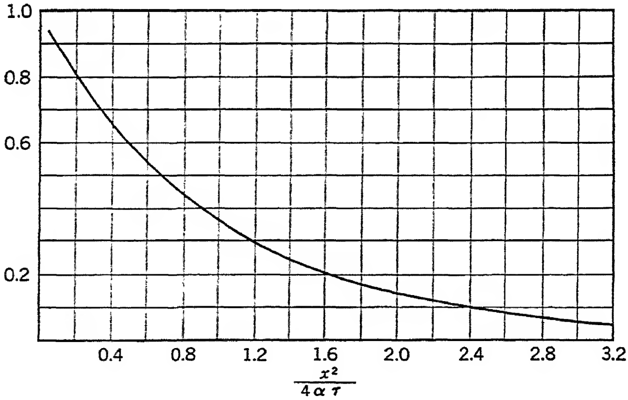


FIG. IV-6. The *e*-function in Eq. IV-19.

The heat energy which has entered the wall through the surface in the whole time τ, beginning with the instant of sudden change of the surface temperature will be

$$Q = \int_{\tau=0}^{\tau=\tau} q \cdot d\tau$$

By substituting q from Eq. IV–20 and by taking all constant fact out of the integral, one obtains

$$Q = \frac{-kA(t_i - t_s)}{\sqrt{\pi\alpha}} \int_{\tau=0}^{\tau=\tau} \frac{1}{\sqrt{\tau}}\, d\tau$$

and by integrating

EXAMPLE IV–3. Calculate the temperature in a plane 7 in. from the su face of a very thick wall, and also the heat flowing into 10 sq ft of this pla 10 hr after the surface temperature of the wall changes suddenly from 70 to 1500 F and remains constant thereafter and also find the total heat energ taken up by the wall in 10 hr. Assume that the average thermal conductivi and diffusivity values of the wall material are 0.5 B hr^{-1} ft^{-1} F^{-1} and 0.0 ft^2 hr^{-1} respectively.

Solution:

$$x \quad - \quad \frac{7/12}{2\sqrt{0.03(10)}} \quad 0.532$$

With this value for $\frac{x}{2\sqrt{\alpha\tau}}$ and according to Fig. IV–5 the value of the func- tion $f\left(\frac{x}{2\sqrt{\alpha\tau}}\right)$ is equal to 0.55. Substituting in Eq. IV–18 gives

$$t = 1500 + (70 - 1500)0.55 = 1500 - 786 = 714 \text{ F}$$

at the required position and time. In order to determine the heat flow, com- pute $x^2/4\,\alpha\tau = 0.284$ and herewith, by referring to Fig. IV–6, find $e^{-x^2/4\alpha\tau} = 0.75$. Substituting the various known values in Eq. IV–19 gives

$$q = -(0.5)10(70 - 1500)\frac{0.75}{\sqrt{\pi 0.03(10)}} = 5520 \text{ B/hr}$$

The total heat energy taken up by the wall is equal to that which has entered the surface. For $\tau = 10$ hr, Eq. IV–21 leads to

$$Q = -(0.5)10\,(70 - 1500)\,2\sqrt{\frac{10}{\pi 0.03}} = 147{,}600 \text{ B}\cdot$$

If the considered wall is of finite thickness L and $\frac{L}{2\sqrt{\alpha\tau}} > 0.6$, the same equations as those for the infinitely thick wall may be used.

IV-4 Sudden Change of the Surface Temperature of a Sphere or Cylinder

For a sphere or a cylinder of diameter D, originally at temperature t_i, throughout, and sudden change of the surface temperature to a perma-

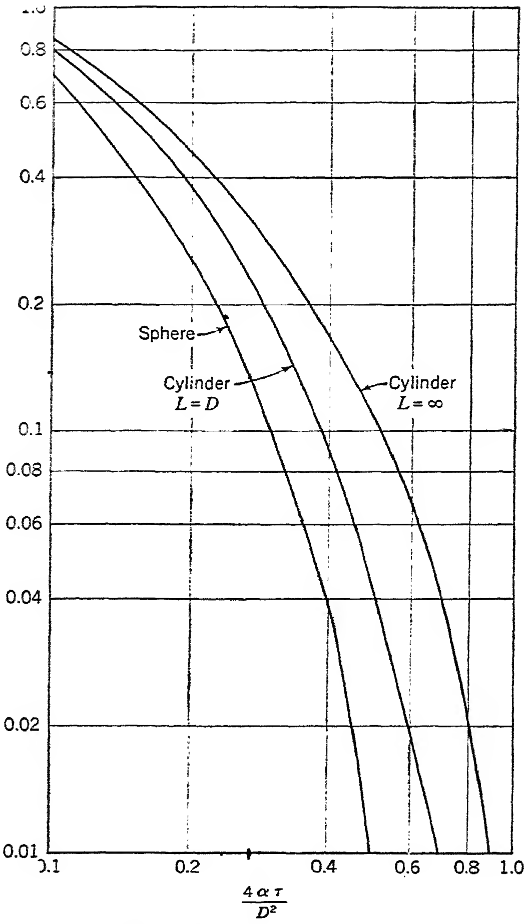


FIG. IV-7. Function F of Eq. IV-22.

nent value t_s, Williamson and Adams (Ref. IV-1) presented the following equation:

$$t_c = t_s + (t_i - t_s)\,F\left(\frac{4\alpha\tau}{D^2}\right) \qquad [IV-22]$$

In this equation t_c is the temperature in the center of the sphere, or the center line of the cylinder, τ hours after the sudden change of the surface temperature. Any value of the function $F\left(\frac{4\,\alpha\tau}{D^2}\right)$ may be obtained from Fig. IV–7 as ordinate at the abscissa $4\,\alpha\tau/D^2$. In this figure curves are given for spheres and cylinders of length $L = D$ and $L = \infty$. For a cylinder of finite length $L > D$, an interpolation will give approximate values.

EXAMPLE IV–4. Calculate the temperature at the center of a steel sphere 16 in. in diameter 20 minutes after the surface temperature changes from 80 F to 1200 F. An average value $\alpha = 0.31$ for steel may be assumed.

Solution:

$$\frac{4\,\alpha\tau}{D^2} = \frac{4(0.31)(20/60)}{(16/12)^2} = 0.23$$

From Fig. IV–7

Substituting in Eq. IV–22, the final temperature at the center is found to be

$$t_c = 1200 + (80 - 1200)\,0.20 = 976 \text{ F}$$

IV–5 Periodical Change of Surface Temperature

A thick plane wall where the surface temperature changes according to the sin-function, as shown in Fig. IV–8, will now be considered. Owing to the resistance of the wall material to the flow of heat, a definite period of time will be required before the surface temperature change affects the temperature distribution within the body. The time lag or the time required for the temperature at a given point within the body to be influenced by a surface temperature change may be found from the following relation:

$$\alpha\pi n$$

In this relation x represents the distance from the surface, n the number of complete changes per unit of time, and α the thermal diffusivity of the material. In Fig. IV–8 abscissas are values of time τ, and ordinates are temperatures. A surface-temperature curve is drawn in full lines, and the temperature at a distance x from the surface is shown by the dotted curve. The time lag $\Delta\tau$ is represented as the distance between two corresponding positions on both curves. From Eq. IV–23 it is seen that $\Delta\tau$ becomes larger the deeper the heat wave enters into the

wall, and that it decreases with increasing frequency n and thermal diffusivity α.

The maximum deviation $\Delta t_{x,m}$ of the temperature from its average at the position x within the body, may be found by use of the following relation:

$$\Delta t_{x,m} = \Delta t_{0,m} \cdot e^{-x\sqrt{\pi n/\alpha}} \qquad \text{[IV-24]}$$

where $\Delta t_{0,m}$ represents the surface-temperature amplitude. From Eq. IV-24 and Fig. IV-8 it is seen that the temperature oscillates between $t_a + \Delta t_{x,m}$ and $t_a - \Delta t_{x,m}$ where t_a is its average value. The

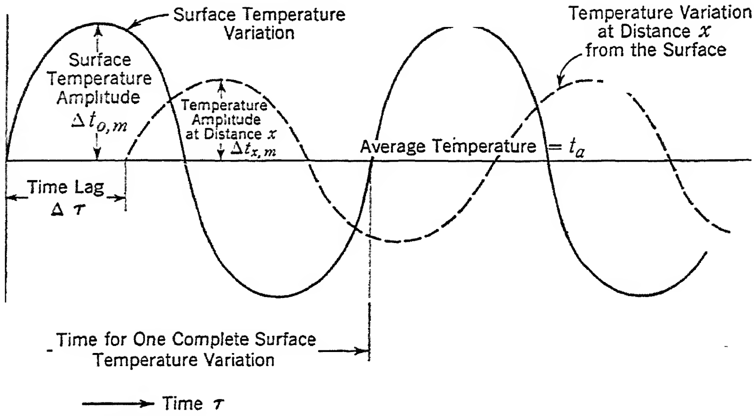


FIG. IV-8. Temperature variation inside a wall for sin-variation of surface temperature.

amplitude of oscillation $\Delta t_{x,m}$ decreases with increasing distance x from the surface. The greater the thermal diffusivity α of the material and the smaller the frequency n, the higher will be the value of $\Delta t_{x,m}$.

The temperature at any point x within the body after τ hours may be calculated from

$$\Delta t_x \qquad \qquad \sqrt{\frac{\pi n}{\alpha}}\Bigg) \qquad \text{[IV-25]}$$

This actually is a combination of Eqs. IV-23 and 24.

Surface temperature variations which approximate this condition are found in the field of heating, ventilating, and air conditioning. Because this particular type of temperature variation is important, several examples will be presented.

EXAMPLE IV–5. A very thick brick wall is heated and cooled periodic every twenty-four hours according to a sin-variation of the temperature. the surface temperature range is from 80 F to 150 F, calculate the time of the temperature wave at a point 8 in. from the surface. Assume t $k = 0.6$ B $\text{hr}^{-1}\,\text{ft}^{-1}\,\text{F}^{-1}$, $c_p = 0.25$ B $\text{lb}_m^{-1}\,\text{F}^{-1}$, and $\gamma = 110\,\text{lb}_f\,\text{ft}^{-3}$.

Solution: Substituting $x = 8/12$ ft, $\alpha = \dfrac{k}{C_m} = \dfrac{0.6}{(110)(0.25)}$ ft²/hr and n = 1/24 in Eq. IV–23 give

EXAMPLE IV–6. If the range in temperature during twenty-four hours at the earth's surface in a locality is −15 F to 25 F, calculate the range temperature at depths (*a*) of 2 ft and (*b*) of 6 ft and the lag of the temperature wave at these depths if a sin-curve variation in surface temperature exists.

Assume that $k = 1.3$ B $\text{hr}^{-1}\,\text{ft}^{-1}\,\text{F}^{-1}$, $c_p = 0.4$ B $\text{lb}_m^{-1}\,\text{F}^{-1}$, and $\gamma =$ 126.1 $\text{lb}_f\,\text{ft}^{-3}$

Solution: $25 - (-1$ $= 20$ F; $n = 1/24$; $\alpha = \dfrac{k}{C} =$

(126.1) (0.4) 0.0258 ft²/hr; $t_a =$ $= 5$ F;

(*a*) $-2\sqrt{\frac{\pi}{0.0258}\cdot\frac{1}{24}}$ 0.22 F

(*b*) 0.00 F

Thus, at a depth of 2 ft the temperature range is from 4.78 F to 5.22 F and at 6 ft the temperature is constant and equal to 5.0 F.

$$\Delta\tau = \frac{2}{2}\sqrt{\frac{24}{0.0258\pi}} = 17.2 \text{ hr}$$

$$\Delta\tau = \frac{6}{2}\sqrt{\frac{24}{0.0258\pi}} = 51.6 \text{ hr}$$

EXAMPLE IV–7. The twenty-four-hour range in temperature at the earth's surface in a given locality is from −10 F to 10 F. Determine (*a*) the amplitude of temperature oscillation at a depth of 1 ft, (*b*) the time lag of the temperature wave at a depth of 1 ft, and (*c*) the temperature at a depth of 1 ft, five hours after the surface temperature reaches the minimum temperature. Assume $k = 0.3$ B $\text{hr}^{-1}\,\text{ft}^{-1}\,\text{F}^{-1}$, $c_p = 0.47$ B $\text{lb}_m^{-1}\,\text{F}^{-1}$ and $\gamma = 100\,\text{lb}_f\,\text{ft}^{-3}$.

Solution:

$$t_a = -\frac{10 + 10}{2} = 0 \text{ F}$$

$$\Delta t_{0,m} = \frac{10 - (-10)}{2} = 10 \text{ F}$$

$$n = \frac{1}{24}$$

$$\alpha = \frac{0.3}{100(0.47)} = 0.00638 \text{ ft}^2/\text{hr}$$

(a) $$\Delta t_{x,m} = 10\, e^{-1\sqrt{\frac{\pi}{0.00638}\cdot\frac{1}{24}}} = 10\, e^{-4.53} = \mathbf{0.11\ F}$$

(b) $$\Delta\tau = \frac{1}{2}\sqrt{\frac{24}{0.00638\pi}} = 17.3 \text{ hr}$$

(c) By substituting $x = 0$ in Eq. IV–25 one obtains the temperature deviation from the average $\Delta t_0 = \Delta t_{0,m} \sin 2\pi n\tau$, and from this $\Delta t_0 = 0$ for $\tau = 0$ in conformance with Fig. IV–8. After that, the minimum temperature on the surface occurs when $\Delta t_0 = -\Delta t_{0,m}$ and this occurs the first time when $2\pi n\tau_m = \frac{3}{2}\pi$ or at $\tau_m = \dfrac{3}{4n} = \dfrac{3(24)}{4} = 18$ hr. Five hours later $\tau = 18 + 5 = 23$. At this time and at $x = 1$ ft according to Eq. IV–25

$$\Delta t_x = 10\, e^{-4.53} \sin\left(\frac{2\pi 23}{24} - 4.53\right) = 10(0.011) \sin 1.49$$

$$= 0.11 \sin 85°6' = 0.11(0.996) = 0.11 \text{ F}$$

The temperature $t_x = t_a + \Delta t_x = 0 + 0.11 = 0.11$ F

PROBLEMS

IV–1. A reinforced concrete foundation slab 28 in. thick is to be poured into a foundation form, the bottom of which is earth at a temperature of 10 F. If the concrete is originally at a temperature of 65 F, calculate approximately the distance the freezing temperature will advance before the slab sets. Assume that the initial set takes place in sixty minutes, the final set in ten hours, and the heat generated in the concrete during setting is neglected. Assume that $k = 0.74$ B hr^{-1} ft^{-1} F^{-1}, $c_p = 0.211$ B lb$_m^{-1}$ F^{-1}, and $\gamma = 141.7$ lb$_f$ ft^{-3}.

IV–2. Calculate the temperature which the center of a steel sphere 0.6 ft in diameter will have attained at a time five minutes after the sphere has been plunged into a heat-treating bath at 100 F. Assume that the surface of the sphere immediately reaches the temperature of the bath. Assume the initial temperature of the sphere to be 1100 F. Assume that $k = 21.5$ B hr^{-1} ft^{-1} F^{-1}, $c_p = 0.132$ B lb$_m^{-1}$ F^{-1}, and $\gamma = 490$ lb$_f$ ft^{-3}.

IV-3. Calculate the temperature at the center of a steel sphere 12 in. in diameter ten minutes after the surface temperature changes from 80 F to 500 F. Assume $\alpha = 0.30 \text{ ft}^2 \text{ hr}^{-1}$.

IV-4. Calculate the temperature at the center of a long cylinder 18 in. in diameter after being placed in the air at 70 F for one hour. Assume that the initial temperature of the cylinder was 1000 F and that the surface temperature immediately reached the temperature of the air. Assume that $k = 22 \text{ B hr}^{-1} \text{ ft}^{-1} \text{ F}^{-1}$, $c_p =$ 0.14 B $\text{lb}_m^{-1} \text{ F}^{-1}$, and $\gamma = 490 \text{ lb}_f \text{ ft}^{-3}$.

IV-5. The wall of a large furnace is made up of refractory material 21 in. thick having a thermal conductivity of $0.7 \text{ B hr}^{-1} \text{ ft}^{-1} \text{ F}^{-1}$ and a diffusivity value of 0.02 $\text{ft}^2 \text{ hr}^{-1}$. A thermocouple 3 in. from the inside surface indicated a temperature of 1250 F seven hours after the firing-up period. If the gas temperature on the inner surface is 2000 F, calculate the theoretical temperature at the thermocouple position. Assume that the innermost layer of the refractory assumes the temperature of the hot gases immediately after the firing-up period begins. The initial temperature of the wall is assumed to be at 65 F.

IV-6. The twenty-four-hour range in temperature at the earth's surface in a given locality is from -10 F to 10 F. Determine (*a*) the range in temperature at a depth of 1 ft, (*b*) the time lag of the temperature wave at a depth of 1 ft, and (*c*) the temperature at a depth of 1 ft, five hours after the surface temperature reaches the minimum temperature. $k = 1.34 \text{ B hr}^{-1} \text{ ft}^{-1} \text{ F}^{-1}$, $c_p = 0.2 \text{ B lb}_m^{-1} \text{ F}^{-1}$, and $\gamma = 126 \text{ lb}_f \text{ ft}^{-3}$.

IV-7. Calculate the temperature of the earth 2 ft below a large rectangular gas conduit after fourteen days from the time the hot gases start to flow through the duct. Assume that the gas temperature is 800 F and the initial temperature of the soil is 70 F. $k = 1.4 \text{ B hr}^{-1} \text{ ft}^{-1} \text{ F}^{-1}$, $c_p = 0.2 \text{ B lb}_m^{-1} \text{ F}^{-1}$, and $\gamma = 126 \text{ lb}_f \text{ ft}^{-3}$.

IV-8. In a recently published article (Ref. IV-2) experimental results are reported on the time lag of the temperature wave through a common brick wall 12½ in. thick. The actual time required for the temperature wave to advance from a point ½ in. from the outer surface to a position ½ in. from the inner surface was nine hours. Using a diffusivity value of $0.0194 \text{ ft}^2 \text{ hr}^{-1}$, check the experimental time lag of the temperature wave by use of the method suggested in this text.

If the range in temperature (defined as twice the amplitude) is 8 F at a plane 11¾ in. from the outer face of the wall, calculate the temperature at a plane 11¾ in. from the outer face after fourteen hours from the time the plane under consideration reaches the maximum temperature. Assume that the mean temperature of the plane equals 76 F.

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IV-2. F. E. Giesecke, "The Flow of Heat through Walls," *Heating, Piping and Air Conditioning*, **10**, 802 (1938).

CHAPTER V

STEADY STATE HEAT CONDUCTION IN BODIES WITH HEAT SOURCES

V-1 Temperature Distribution in a Plane Plate in Which Heat is Produced Homogeneously

The transient storage of heat energy has been considered in Chapter IV. The subject of heat sources within a heat-conducting body will now be discussed. Cases of this type occur in exothermic chemical reactions such as the combustion of fuel in the fuel bed of a boiler furnace. Another example is an electrical coil wherein heat is produced and is dissipated by thermal conduction.

Only the simplest type of this process will be dealt with in this text. Figure V-1 shows a perspective sketch of a vertical section taken through a cylindrical coil. The rate of heat produced homogeneously in the electric wires per unit volume of the coil may be calculated from Ohm's law. This unit volume includes the volume occupied by the insulation and the wires. For purposes of simplification the height of the coil will be assumed to be such that only heat conducted in radial direction must be considered. If the influence of the curvature of the coil is negligible, then the coil shown in Fig. V-1 may be replaced by a plane plate with heat sources uniformly distributed inside the plate (Fig. V-2). Here heat will be conducted only in the directions $+x$ and $-x$. The two surfaces will be assumed at a constant temperature t_s and are represented by $x = L$ and $x = -L$.

When no heat sources existed, the heat energy stored in unit time in a slice of the plate of thickness dx was found to be $kA \dfrac{\partial^2 t}{\partial x^2}\, dx$ according to Eq. IV-6. If the rate of heat q''' per unit volume* is produced in the coil an amount $q'''(A \cdot dx)$ will likewise be stored. However, according to Eq. IV-15 the total rate of heat storage under steady state conditions must be zero. The following equation results.

$$kA \frac{\partial^2 t}{\partial x^2}\, dx + q''' A \cdot dx = 0$$

or

$$k \frac{d^2 t}{dx^2} + q''' = 0 \qquad \text{[V-1]}$$

* In this text q' is in B hr^{-1} ft^{-1}, q'' is in B hr^{-1} ft^{-2}, and q''' is in B hr^{-1} ft^{-3}.

where k represents the heat conductivity of a fictitious substance having the same composition of insulation and wire as an actual volume of the coil. The value of k may easily be measured or calculated. The total differential is used in Eq. V–1 because in steady state conditions the

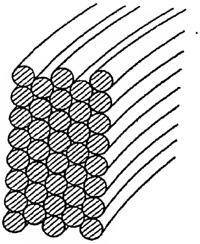

FIG. V–1. Section through a cylindrical coil.

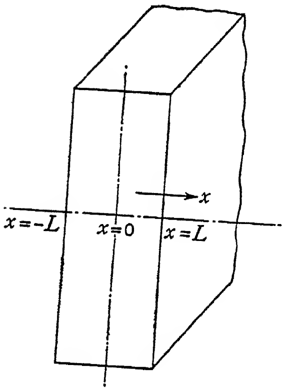

FIG. V–2. Plane plate as simplified representation of a cylindrical coil.

temperature t does not depend on the time τ, but only on x. If $\theta = t - t_s$ is introduced in Eq. V–1, that is, counting the coil temperature from the surface temperature as zero point, the equation is converted into the form

$$k\frac{d^2\theta}{dx^2} + q''' = 0 \qquad \text{[V–2]}$$

Since t_s is supposed to be constant, $d^2\theta/dx^2 = d^2t/dx^2$. Integration of Eq. V–2 gives

$$\theta = -\frac{q'''}{2k}x^2 + Mx + N \qquad \text{[V–3]}$$

The constants of integration M and N may be found from two boundary conditions. The maximum temperature, θ_m, in the coil will occur at the mid-plane. So at $x = 0$, $\theta = \theta_m$. Substituting this in Eq. V–3 gives

$$N = \theta_m \tag{V–4}$$

θ must decrease symmetrically in the direction of x and $-x$ from the maximum value. This condition may only be satisfied if in Eq. V–3,

$$M = 0 \tag{V–5}$$

Substituting Eqs. V–4 and 5 in Eq. V–3 gives

$$\theta = -\frac{q'''}{2k}x^2 + \theta_m \tag{V–6}$$

Since $t = t_s$ or $\theta = 0$ at $x = L$ and at $x = -L$

$$\theta_m = \frac{q'''}{2k}L^2$$

which when substituted in Eq. V–6 gives

$$\theta = \theta_m\left(1 - \frac{x^2}{L^2}\right) \tag{V–7}$$

This means that a parabolic temperature distribution exists across the cross section of the coil with the apex at the median plane.

V–2 Maximum Temperature in an Electric Coil

According to Vidmar (Ref. V–1) there exists a simple and practical relation between the maximum and average temperature of a coil.

The electrical resistance of copper and other pure metal increases linearly with the temperature. For the sum of all the lengths of wire inside a slice of unit thickness of the plate

$$R' = R_s'(1 + \epsilon\theta) \tag{V–8}$$

where

R' = the resistance of the sum of the wire lengths at a temperature t
R_s' = the same at a temperature t_s, and
ϵ = the temperature coefficient of electric resistance of the metal wire. ($\epsilon = 0.0024$ F^{-1} for copper.)

For a slice of thickness dx, then

The rate of heat energy produced in the coil by a current I in the coil

according to the laws of Ohm and Joule is

$$q'''A \cdot dx = I^2R'dx = I^2R_s'(1 + \epsilon\theta)dx$$

Integrating over the thickness of the coil

$$q = q'''A2L = \int_{x=-L}^{x=L} I^2R' \cdot dx = \int_{x=-L}^{x=L} I^2R_s'(1 + \epsilon\theta)dx \qquad \text{[V-9]}$$

In electrical engineering the average temperature θ_a of a coil when a current I is flowing is found by measuring its electrical resistance R_a and also measuring its resistance R_s at a very small current such that the coil does not become heated sensibly above t_s. This gives another relation for the heat produced electrically in the coil.

$$q = I^2R_a = I^2R_s(1 + \epsilon\theta_a) = I^2R_s'2L(1 + \epsilon\theta_a) \qquad \text{[V-10]}$$

In this equation $R_s = R_s'2L$, since R_s' was defined as the resistance for the thickness 1, and the coil is $2L$ thick.

Equating q from Eqs. V–9 and 10 it follows that

$$\theta_a = \frac{1}{2\,L}\int_{x=-L}^{x=L} \theta dx$$

Substituting θ from Eq. V–7 gives

$$\theta_a = \theta_m \int_{x=-L}^{x=L} \left(1 - \frac{x^2}{L^2}\right) dx = \theta_m - \frac{\theta_m}{3} = \frac{2}{3}\,\theta_m$$

or

$$\frac{\theta_a}{\theta_m} = \frac{t_a - t_s}{t_m - t_s} = \frac{2}{3} \qquad \text{[V-11]}$$

or

$$t_m = t_s + \tfrac{3}{2}\,(t_a - t_s) \qquad \text{[V-12]}$$

Thus by measuring the surface temperature t_s of the coil by means of an attached thermocouple and by determining the average temperature t_a or $\theta_a = t_a - t_s$ from the increase of electrical resistance when a current is passing through the coil, the maximum temperature t_m inside the coil at that current may easily be found. This temperature by which the durability of a coil is limited can scarcely be measured by any other way in practice. Placing thermocouples inside the coil is rarely possible owing to the difficulty of electric insulation of the thermocouples against the main current.

Example V–1. A vertical cylindrical electric coil 1¾ in. thick having an inside diameter of 10 in. is heat insulated on the bottom and top so that no appreciable amount of heat will flow in a vertical direction.

When located in an oil bath the surface temperature of the coil was found to be 70.5 F at a current and voltage of 0.009 amp and 0.1 volt respectively. At 2.5 amp the resistance of the coil was 12.1 ohms. However, the surface temperature remained unchanged. Calculate the maximum temperature inside the coil when the current was 2.5 amp. Assume that the wires were made of copper.

Solution: The ratio of the radii 10/13.5 of the coil is so ~~great~~ that the curvature may be neglected. A current of 0.009 amp is negligible as compared with 2.5 amp because by the latter according to Ohm's law $\left(\dfrac{2.5}{0.009}\right)^2 = 77{,}000$ times as much heat is produced than by the former one. Thus the starting temperature of the entire coil was $t_s = 70.5$ F. According to Eq. V–10

$$R_a = R_s(1 + \epsilon\theta_a)$$

$$R_s = \frac{0.1 \text{ volt}}{0.009 \text{ amp}} = 11.1 \text{ ohm}$$

$$\epsilon = 0.0024 \text{ F}^{-1}$$

$$R_a = 12.1 \text{ ohm}$$

Thus

$$12.1 = 11.1(1 + 0.0024\theta_a)$$

and

$$\theta_a = \frac{0.09}{0.0024} = 37.4 \text{ F}$$

According to Eq. V–11

$$\theta_m = 1.5\theta_a = 56.1 \text{ F}$$

Therefore, the maximum temperature of the coil was

$$t_m = t_s + \theta_m = 70.5 + 56.1 = 126.6 \text{ F}$$

REFERENCE

V–1. M. VIDMAR, "Suggestion of an Addition to the Test Codes on Temperature Rise,"* *Elektrotechn. u. Maschinenbau*, **26**, 49, 65 (1918).

* Title translated by the authors.

CHAPTER VI

INTRODUCTION TO THE DIMENSIONAL ANALYSIS OF CONVECTION

VI–1 The Nature of Heat Convection

Heat conduction is considered as a molecular exchange of energy, and therefore it is governed by molecular laws, some of which have been mentioned in Chapter II. The motions of molecules involved in this process cannot be seen by our eyes nor measured by ordinary hydraulic or aerodynamic instruments. In contrast to this, heat convection is a transportation of heat energy by fluids in ordinary motion which is either directly visible or can be visualized by the usual instruments for the measurement of the flow of fluids. Heat transfer by convection is the exchange of heat energy between moving parts of the fluid or between these and surfaces of different temperature.

Since in this process heat is conveyed mechanically from one particle to another, it is obvious that the transfer of energy depends upon the motion of the fluid and is governed by the laws of fluid dynamics, in addition to the laws of heat conduction and heat storage which must be considered at the same time.

From this it will be understood that heat convection is a very complex process and that the simplicity of Eq. I–3 is delusive. As a matter of fact, the film coefficient h, defined by that equation, is a function of many variables, such as shape and dimensions of the surface, kind, direction, and velocity of the flow, temperature, density, viscosity, specific heat, and thermal conductivity of the fluid.

The differential equations which describe convective heat transfer belong to the most difficult class in theoretical physics, and only for a very few simple cases has it been possible to solve them under simplifying assumptions.

The empirical treatment proved to be likewise unsatisfactory. Because of the many variables mentioned above, it was almost impossible to find any more general relations, i.e., equations which could be applied to other arrangements and conditions than just those used by a particular investigator.

Thus, the knowledge of heat transfer by convection was extremely meager until about thirty years ago when the application of the so-called

principle of similarity to heat transfer problems began to change the situation entirely.

Before explaining this principle and showing how it is used in problems of heat transfer, a few remarks and definitions about the flow of fluids will be presented.

VI–2 Reynolds' Concept of Similarity of the Flow of Fluids and the Viscosity

It has been known for a long time that two general types of motion may take place in the flow of a fluid through a pipe, each following a different law. Whenever the fluid particles flow in paths that are parallel to the axis of the pipe without radial components, the flow is called viscous, laminar, or streamlined. The condition where radial components or eddies exist together with the fluid motion parallel to the pipe axis is known as turbulent flow. Osborne Reynolds investigated the problem of the " similar " flow of fluids. In this study he considered the circumstances which would make the flow of different fluids through circular tubes of different diameters with different velocities " similar." In a famous paper published in 1883 (Ref. VI–1) he showed that the flow of fluids is similar when a certain " dimensionless " group of variables is the same. This group, which in his honor now is called Reynolds number,* is

$$(Re) = \frac{v_a D\gamma}{\mu g} \qquad \text{[VI–1]}$$

In these ratios D represents the diameter, v_a the average fluid velocity, and ρ, γ, and μ the density, specific weight, and dynamic viscosity of the fluid respectively.

The dynamic viscosity μ is defined by Newton's equation

$$F_s = \quad dy \qquad \text{[VI–2]}$$

where F_s is the shearing force or fluid friction between two parallel layers of a fluid, which have equal areas A, are separated by the distance dy, and are moving, one parallel to the other, with the velocities of v and $v + dv$ respectively. According to this definition dv/dy is the velocity slope or gradient perpendicular to the direction of the flow of the two fluid layers. If $A = 1$ and $dv/dy = 1$, $F_s = \mu$.

* The two first letters of Reynolds' name are in general used as a symbol for Reynolds number. It is recommended to use this symbol with parentheses.

The physical dimensions of μ are found by Eq. VI–2 in the same wa[y] as were those for k by Eqs. II–2 and 3.

$$1[\text{lb}] = 1[\mu]\, 1[\text{ft}^2]\, \frac{1[\text{ft sec}^{-1}]}{1[\text{ft}]}$$

or

$$[\mu] = [\text{lb ft}^{-2}\text{ sec}] \tag{VI-3}$$

In referring to Sect. I–3 it must be kept in mind that [lb] in the British system of units is a force. Instead of [lb] one can substitute the mas[s] unit [slug] using the equation of definition

$$1[\text{lb}] = 1[\text{slug}]\, 1[\text{ft sec}^{-2}]$$

and one obtains

$$[\mu] = [\text{slug ft}^{-1}\text{ sec}^{-1}] \tag{VI-4}$$

The unit [sec] can be replaced by [hr/3600] in these equations.

The equation analogous to Eq. VI–4 in the physical system of units i[s]

$$[\mu] = [\text{gram cm}^{-1}\text{ sec}^{-1}] \tag{VI-5}$$

This unit is called a poise in honor of the French physician Poiseuille who was one of the first to state the law of pressure drop in streamline flow in a tube. The unit poise is often used in physics, particularly its hundredth part which is called a centipoise. Water at 68.4 F has a viscosity value of exactly 1 centipoise. Values for water at other temperatures in lb ft^{-2} hr units are given in Table VI–1 and values for air in Table VI–2.

TABLE VI–1

DYNAMIC VISCOSITY OF WATER FOR VARIOUS TEMPERATURES

Temperature F	Dynamic Viscosity* 10^{-9} lb ft^{-2} hr
40	8.99
50	7.59
60	6.49
70	5.66
80	5.00
90	4.43
100	3.96
110	3.58
120	3.25
130	2.97
140	2.73
150	2.51

* Data taken from "International Critical Tables," Vol. 5, 1929.

TABLE VI-2

DYNAMIC VISCOSITY OF AIR FOR VARIOUS TEMPERATURES AT ATMOSPHERIC PRESSURE*

Temperature F	Dynamic Viscosity 10^{-9} lb ft^{-2} hr
40	0.101
50	0.102
60	0.104
70	0.105
80	0.106
90	0.108
100	0.109
110	0.110
120	0.112
130	0.113
140	0.114
150	0.115
160	0.117
170	0.118
180	0.119
190	0.121
200	0.122

* Data taken from A. Eucken and M. Jakob, *Der Chemie-Ingenieur*, Vol. I, Part 1, Chapt. II. Akademische Verlagsgesellschaft, Leipzig, 1933.

The ratio μ/ρ which occurs in Eq. VI–1 is called kinematic viscosity and is denoted by ν. Because the unit of the density ρ is [slug ft^{-3}] = [lb ft^{-1} sec^2 ft^{-3}] = [lb ft^{-4} sec^2], one obtains from Eq. VI–3, as well as from Eq. VI–4,

$$[\nu] = \left[\frac{\mu}{\rho}\right] = [\mathrm{ft}^2/\mathrm{sec}] \qquad \text{[VI–6]}$$

Since in this equation only kinematic magnitudes (length and time), but no dynamic magnitudes (force or mass) occur, the kinematic viscosity is a very convenient name for ν. In contrast to this, μ is called "dynamic viscosity" in this text (Ref. VI–2) instead of the name "absolute viscosity" which seems to be less specific and rather in contrast to "relative viscosity" than to "kinematic viscosity."

Substituting ν from Eq. VI–6 in Eq. VI–1, the latter simplifies to

$$(Re) = \frac{v_a D}{\nu} \qquad \text{[VI–7]}$$

and it is easily shown that the Reynolds number (Re) is really a dimensionless magnitude.

The advantage of using Eq. VI–7 is that neither force nor mass units are involved. Many authors, however, express (Re) in terms of the

so-called mass velocity $M = v_a\rho$, with which Eq. VI–1 converts into

$$(Re) = \frac{MD}{\mu} \qquad \text{[VI-}$$

Using this definition has the advantage that M is independent of pr sure, temperature, and phase of the fluid. Since M has the dimensio [slug ft^{-2} sec^{-1}], μ must be expressed in [slug ft^{-1} sec^{-1}] units accordi to Eq. VI–4.

Because the unit "slug" occurs in both the numerator and denomin tor of Eq. VI–7a, both may be divided by 32.2. Then, since 1/32.2 of slug equals 1 lb$_m$, the following equations of units may be assumed:

$$[M] = [\text{lb}_m\ \text{ft}^{-2}\ \text{sec}^{-1}]$$

and

$$[\mu] = [\text{lb}_m\ \text{ft}^{-1}\ \text{sec}^{-1}] \qquad \text{[VI-7}b\text{]}$$

The latter unit is only 1/32.2 of the value of the unit according to Eq. VI–4 or 3.

If Eq. VI–7b is found in books and articles without the subscript m or "mass" in connection with the unit lb, it must be kept in mind that the pound used therein means the mass of a pound, and the force unit in this system is the so-called poundal, equal to 1/32.2 of the gravitational force which acts on a pound.

However, because μ does not appear alone in Eq. VI–1, but only in the term μ/ρ, a similar conversion is possible as was noted for the product $c_p\rho$ in Sect. IV–1. By multiplying both sides of Eq. VI–7b by g it follows

$$[g\mu] = [\text{lb}_f\ \text{ft}^{-1}\ \text{sec}^{-1}] \qquad \text{[VI-7}c\text{]}$$

Thus, the latter expression is not the unit of μ, as sometimes is believed, but rather the unit of $g\mu$.

On the other hand it is allowable in the numerical calculation of (Re) according to Eq. VI–1, to replace ρ[lb$_m$/cu ft] by the numerical equal value of γ[lb$_f$/cu ft] and μ[lb$_m$ ft^{-1} sec^{-1}] by the numerical equal value of $(g\mu)$ in [lb$_f$ ft^{-1} sec^{-1}]. This procedure simplifies the calculation if μ is given in the units used in Eq. VI–7b as often is the case. In this book however, in general, the unit [lb ft^{-2} hr] or the unit [slug ft^{-1} hr^{-1}] will be used, both belonging to the British technical system with the hour instead of the second as unit of time.

Example VI–1. Calculate the Reynolds number for water at 80 F flowing through a tube 2 in. in diameter, if the average water velocity and specific weight is 10 ft/sec and 62.4 lb/ft^3 respectively.

Solution: According to Table VI–1 the value for the viscosity is 5.00 (10^{-9}) lb ft^{-2} hr. By converting to mass units

$$\mu = 5.00(10^{-9})3600^2 = 0.0648 \text{ slug ft}^{-1}\text{ hr}^{-1}$$

$$\rho = \frac{62.4}{32.2} = 1.94 \text{ slug/ft}^3 \text{ (see Sect. I–3)}$$

Substituting in Eq. VI–6 gives

$$\nu = \frac{0.0648}{1.94} \qquad 0.0334 \text{ ft}^2\text{/hr}$$

Then from Eq. VI–7

It may be noticed that seconds, wherever they occur, are converted into hours.

Example VI–2. If the mass velocity for a fluid at 80 F flowing through a 1-in. diameter tube is 30,000 slug hr^{-1} ft^{-2}, calculate the Reynolds number.

Solution: Substituting in Eq. VI–7*a* gives

Since the velocity distribution is the same if the Reynolds number is the same, it is obvious that the latter is decisive for all characteristics of the flow of a fluid which depend on the velocity distribution, as for eg the pressure drop in a tube.

If, for instance, it is desired to find the pressure drop at twice the Reynolds number of a specified state, this can be accomplished by doubling either the velocity or the pipe diameter. Or if experimental data dealing with the pressure drop for a fluid having a kinematic viscosity ν_1 flowing through a tube of diameter D_1 and velocity v_1 are known, then it is possible to determine the pressure drop for a fluid having a kinematic viscosity $\nu_2 = 2\nu_1$ under the same conditions by conducting experiments with the first fluid using a velocity $v_2 = v_1/2$ or using a pipe having a diameter $D_2 = D_1/2$ because, according to Eq. VI–1, the result leads to the same Reynolds number.

Because the velocity distribution in the cross section depends on the magnitude of (Re), Reynolds further guessed that the transition from the laminar type of flow to the entirely different conditions of turbulent flow might take place at a definite Reynolds number. Indeed his experiments showed that turbulence starts at Reynolds numbers of about 2000 to 2100. These experiments have been repeated frequently; the best known led to $(Re) \approx 2300$. This is called the critical Reynolds number.

VI–3 General Basis of Dimensional Analysis

The usefulness of the Reynolds number for correlating results for the flow of fluids through pipes cannot be ove

A general method by which groups lik attained in different branches of science is basic principle upon which this is founded : that is, dissimilar quantities cannot be added together to form physical relation. Mathematically, this means that in any physical equation both sides of which can be written as power functions or as algebraic sums of power functions, the sum of the exponents of the basic units must be the same on the left and right according to the laws of freely falling bodies,

$$v =$$

the body has fallen through the distance l to gravity. As a numerical example

$$\frac{\;}{\text{sec}^2}\ 14\ \text{ft}$$

Considering the basic units alone

$$[\text{ft}^1\ \text{sec}^{-1}] = [\text{ft}^{1/2}\ \text{sec}^{-1}\ \text{ft}^{1/2}]$$

Obviously, the above rule for the sum of the exponents yields

$$1 = \tfrac{1}{2} + \tfrac{1}{2} \quad \text{for the exponents of ft}$$

and

$$-1 = -1 \quad \text{for the exponents of sec}$$

Now assume that Eq. VI–8 is not known, but that it is only postulated that v depends on g and l. The latter fact may be expressed by

$$v = Cg^a l^b \qquad \text{[VI–9]}$$

where a and b are unknown exponents, and C is an unknown constant.

Representing the fundamental units of length and time by L and T respectively, an equation for the dimensions of Eq. VI–9 is

$$LT^{-1} = C(LT^{-2})^a (L)^b$$

and the rule of the sums of the exponents yields

for L $\qquad 1 = a + b$

for T $\qquad -1 = -2a$

Solving these two equations for a and b

$$a = \tfrac{1}{2}$$

and by substitution in Eq. V-9

$$v = C\sqrt{gL}$$

Therefore, the law, given by Eq. VI-8, has been found by the method of dimensional analysis and only the constant C remains unknown and must be determined by experiments. According to Eq. VI-8, $C = \sqrt{2}$.

VI-4 Application of Dimensional Analysis to Free Convection

Exactly the same method can be applied for the much more complicated cases of heat convection. This was done first by W. Nusselt who in 1909 developed basic equations for forced convection, and in 1915 equations for free convection. The method will be shown by the application of the principle to the example of free convection of a fluid in contact with a warm vertical wall. Before any valid relations may be established, it is essential to know what variables are involved.

Correct selection of the variables comes about through experience. For this particular case, the variables involved are listed together with the dimensions in Table VI-3.

The dimensions of the heat transfer coefficient h can be determined from Eq. I-3 exactly in the same way as those of the heat conductivity k

TABLE VI-3

List of Variables and Their Dimensions for Natural Convection

Variable	Symbol	Exponents of units of				
		Heat	Time	Length	Mass.	Temp.
		H	T	L	M	Θ
Heat transfer coefficient	h	1	−1	−2	0	−1
Height of vertical wall	l	0	0	1	0	0
Temperature difference between the surface and the fluid	Δt	0	0	0	0	1
Density of the fluid	ρ	0	0	−3	1	0
Dynamic viscosity of the fluid	μ	0	−1	−1	1	0
Coefficient of thermal expansion of the fluid	β	0	0	0	0	−1
Acceleration due to gravity	g	0	−2	1	0	0
Specific heat of the fluid	c_p	1	0	0	−1	−1
Thermal conductivity of the fluid	k	1	−1	−1	0	−1

have been derived from Eq. II–1 in Eqs. II–2 and 3. Thus in British technical units

$$[h] = [\mathrm{B\ hr^{-1}\ ft^{-2}\ F^{-1}}] \qquad \text{[VI–]}$$

All other dimensions of Table VI–3 have been derived before or are well known.

A general relation which includes all the variables of Table VI–3 is

$$h = Cl^a(\Delta t)^b\rho^f\mu^i\beta^j g^m c_p^n k^p \qquad \text{[VI–11]}$$

The corresponding equation of dimensions is

$$HT^{-1}L^{-2}\Theta^{-1} =$$

$$CL^a\Theta^b(ML^{-3})^f(MT^{-1}L^{-1})^i\Theta^{-j}(T^{-2}L)^m(HM^{-1}\Theta^{-1})^n(HT^{-1}L^{-1}\Theta^{-1})^p$$

The equations for the sums of the exponents become

$$\begin{array}{ll} H & 1 = n + p \\ T & -1 = -i - 2m - p \\ L & -2 = a - 3f - i + m - p \\ M & 0 = f + i - n \\ \Theta & -1 = b - j - n - p \end{array}$$

By means of these equations, five of the exponents may be expressed in terms of the others, say j, m, n. By simple algebraic operations the following is obtained:

$$a = -1 + 3m$$

$$b = j$$

$$f = 2m$$

$$i = -2m + n$$

$$p = 1 - n$$

By substituting these five exponents in Eq. VI–11, rearranging and collecting the terms containing like exponents

$$\frac{hl}{k} = C\left(\frac{l^3\rho^2 g}{\mu^2}\right)^m (\Delta t \cdot \beta)^j\left(\frac{\mu c_p}{k}\right)^n \qquad \text{[VI–12]}$$

This equation can be further simplified if the relation between g and β is considered. Free convection is due to a thermal buoyancy effect. If in a fluid at temperature t_1 and density ρ_1 a unit of volume is heated to temperature t_2, and, as a result, has a smaller density ρ_2, then according

to Archimedes' law, the buoyancy $B = \rho_1 g - \rho_2 g$ acts upon the mentioned unit of volume. However, with respect to the definition* of the coefficient of thermal expansion $\rho_1 = \rho_2[1 + \beta(t_2 - t_1)]$ and by substitution $B = \rho_2 \beta g(t_2 - t_1)$. From this, since β and g enter the equations owing only to the buoyancy, they will always appear in the form of a product (βg). According to Eq. VI–12 this is possible only if $j = m$. Therefore, it simplifies

$(Gr)^m \; (Pr)^n$

$$\frac{hl}{k} = C\left(\frac{l^3\rho^2\beta g \cdot \Delta t}{\mu^2}\right)^m \left(\frac{\mu c_p}{k}\right)^n = C\left(\frac{l^3\beta g \cdot \Delta t}{\nu^2}\right)^m \left(\frac{\mu c_p}{k}\right)^n \qquad \text{[VI–13]}$$

(Gr) (Pr)

For ideal gases it can be proved exactly that $\beta = 1/T_1$, where T_1 is the absolute temperature of the main bulk of the gas.†

It may be mentioned that as early as 1881, L. Lorenz by integration of the differential equations of fluid flow and heat convection came to an equation for free convection of gases on a vertical wall which is a special case of Eq. VI–13 with $m = n = \frac{1}{4}$. The constant C was determined by Lorenz for different gases; it was found to be an individual constant for each of them.

It may further be mentioned that Eq. VI–13 holds also for cylinders (wires or tubes) located horizontally in a fluid at rest. For these l is the diameter of the cylinder.

VI–5 Main Advantage of Dimensional Analysis

Examination shows that each of the three groups of magnitudes in this equation is dimensionless, like Reynolds number, which as explained, has the advantage that one can change any involved magnitudes together with the other magnitudes in such a way that the numerical value of the dimensionless group remains the same. By this the result of any variation of each of these magnitudes can be obtained by simple calculations if the influence of the change of *only one* magnitude in a group is found from experimental investigation. The three constants C, m, and n must be determined by experiments.

What work and time may be spared by setting up an equation of dimensionless groups like Eq. VI–13 can be seen by the following estimation: Assume that in order to find h for all thinkable variations, one would proceed by changing first l, taking, for instance, five different cylinder diameters, then conducting experiments with each of these and with fluids of five different thermal conductivities k, further varying ρ five times, and so on each of the seven independent variables l, k, ρ, β, Δt, μ, c_p. This would mean $5^7 = 78{,}125$ tests. According to Eq. VI–13

* This definition is slightly different from Nusselt's original definition.

† This does not hold for Nusselt's original definition of β.

on the other hand it would be sufficient to make experiments with fi- different values of the first group at the right, each with five values of th- second group, that is, $5^2 = 25$ experiments, in order to get about th- same result as with more than 78,000 tests with the direct method.

Actually, of course, more tests than the minimum of 25 would b- conducted; on the other hand, not all the variations of the mention- seven independent variables would be found necessary or even possibl- However, the saving of work and time would still be enormous. As matter of fact, thousands of random experiments conducted before with- out the use of the principle of similarity have not led to any reliabl- knowledge of the behavior of heat transfer, whereas nowadays, relativel- small numbers of tests yield very elucidating results.

However, it must not be forgotten that simplifying assumptions are inevitable in this method. For instance, since the temperature changes across the fluid, the same holds for the viscosity and thermal conductiv- ity which depend on the temperature; but it is almost impossible to con- sider this influence exactly in the dimensional analysis. For this and other reasons the results of dimensional analysis may not be very exact and often only show a qualitative agreement with experience.

Similar dimensionless ratio equations as Eq. VI–13 have been work out for many cases of heat transfer and are extremely useful for solvi problems.

VI–6 The Most Important Dimensionless Groups

Dimensionless groups occur over and over again. The most importar are the following:*

Reynolds number $(Re) = \dfrac{vD\rho}{\mu}$ or $\dfrac{vD\gamma}{\mu g}$ or $\dfrac{vD}{\nu}$ or $\dfrac{MD}{\mu}$

Grashof number $(Gr) = \dfrac{D^3\rho^2\beta g \cdot \Delta t}{\mu^2}$

Prandtl number $(Pr) = \dfrac{\mu c_p}{k}$ or $\dfrac{\nu}{\alpha}$

Nusselt number $(Nu) = \dfrac{hD}{k}$

Here a diameter D is used as a characteristic length. In the case of a vertical wall, for instance, the height must be taken instead of this in the Grashof number (see Eq. VI–13).

It may be mentioned that the Prandtl number depends only on the physical properties of the fluid, and therefore itself is a physical property

* As (Re), also the other dimensionless groups in general are denoted by the two first letters of the name of an investigator who is to be honored by the denotation.

of the flowing substance, whereas the other groups depend also on the geometrical arrangement, the flow, the temperature difference, etc.

With the symbols, given above, Eq. VI–13 may be written in the abbreviated form

$$(Nu) = C(Gr)^m(Pr)^n \qquad [\text{VI–14}]$$

The use of dimensional analysis for forced convection will be dealt with in Chapter VIII.

PROBLEMS

VI–1. Verify the following statement: Divide the surface film coefficient in $\text{B hr}^{-1}\,\text{ft}^{-2}\,\text{F}^{-1}$ units by 0.2048 to convert it to $\text{kcal hr}^{-1}\,\text{m}^{-2}\,\text{C}^{-1}$.

VI–2. Verify by dimensional analysis the equation given below for the heat transferred from a rigid body submerged in a flowing fluid. It is to be assumed that the body is maintained at a definite temperature which is higher than the fluid temperature. The variables other than the rate of heat transfer (q) are a linear dimension of the body (l), the velocity of the fluid, (v), the temperature difference between the body and the fluid (Δt), the specific heat of the fluid based on a unit volume (C_p), and the thermal conductivity of the fluid (k).

$$\left(\frac{lk \cdot \Delta t}{q}\right) = C\left(\frac{lC_p v}{k}\right)^a$$

VI–3. Verify the equations given below which have been used for correlating the data for heat transfer for fluids flowing in turbulent motion through the inside of pipes.

$$(Nu) = C(Re)^a(Pr)^b$$

VI–4. What is the significance of G in the following equation of definition of the Reynolds number:

$$g\mu$$

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* Title translated by the authors.

CHAPTER VII

HEAT TRANSFER BY FREE CONVECTION

VII–1 Some Peculiarities of Free Convection

As already explained, the motion of the fluid particles in natural or free convection heat transfer is due to the differences in densities of the parts of the fluid. A domestic gravity hot-air heating system is an example of heat transfer by natural convection. Heated air rises through the furnace pipes, passes through the registers, is cooled at the walls, windows, etc., of the room, and then returns through the cold-air ducts to the furnace. Another example of natural convection is the heat transferred from the surface of an insulated pipe to the still air of a room. In this case, some of the heat is transferred by radiation. On the other hand, the transfer of heat from hot flue gases to the tubes of an economizer or water heater serves as an example of forced convection, since the flue gases are made to pass through the unit by fans.

In Chapter VI it has been shown that natural convection heat transfer theory is complicated because of the many variables involved in the solution of a particular problem. It has also been mentioned that a complete mathematical solution is known for only a very few cases.

The experimental data on natural convection for some cases have been correlated by dimensional analysis and these correlations are useful in the solution of problems. Approximate simplified equations which apply to a limited range of experimental data have been developed for several cases. Some of these equations will be given in this chapter. They should be used with considerable reservation since they are valid only for a limited range.

There is another item which requires consideration. Δt in the basic Eq. I–3 is called the temperature difference between the area A of the surface and the fluid in contact with it. The temperature of the surface is well defined and can be measured by attached or inserted thermocouples. However, there is no unique temperature of the fluid. According to experience, the temperature of the fluid is equal to the surface temperature at the places where the fluid is in contact with the surface; then, if the surface is warmer than the environment, the temperature decreases sharply in a thin fluid layer close to the surface and finally it becomes almost constant at some distance from the wall. If the surface

is cooler than the environment, the behavior is similar, with a sharp temperature increase of the fluid beginning at the surface. In natural convection, Δt generally is defined as the difference between the temperature of the surface and that of the fluid at some distance, say several feet, from the surface.

VII-2 Empirical Equations for Heat Transfer on Horizontal and Vertical Surfaces

The following empirical equations (Ref. VII-1) can be used in the approximate solution of problems of heat transfer between horizontal plates and still air under free convection conditions.

$$h = 0.38(\Delta t)^{0.25} \tag{VII-1}$$

for plates facing upward and

$$h = 0.2(\Delta t)^{0.25} \tag{VII-2}$$

for plates facing downward.

It may be mentioned that Δt here stands for Δt degree Fahrenheit/1 degree Fahrenheit, that is, a dimensionless number which in the metric system would be the same, namely $\Delta t/(5/9)$ where Δt is in degrees centigrade. The constants 0.38 and 0.2, however, are not dimensionless. The corresponding constants for the metric system would be 4.88 (0.38) and 4.88 (0.2), respectively.

Example VII-1. Calculate the heat loss by natural convection from a horizontal heated plate at 300 F facing upward to the still air of a room at 80 F. The plate is 7 in. by 9 in.

Solution:

$$h = 0.38(300 - 80)^{0.25} = 1.47 \text{ B hr}^{-1}\text{ ft}^{-2}\text{ F}^{-1}$$

The total loss is

$$q = 1.47 \frac{(7)(9)}{144}(300 - 80) = 142 \text{ B/hr}$$

For calculating approximately the heat transfer by free convection between vertical plates and still air the following formulas may be

for plates of height $L > 1$ ft, and

$$h = 0.28\left(\frac{\Delta t}{L}\right)^{0.25}$$

for plates of height $L < 1$ ft.

Example VII–2. Calculate the film coefficient for free convection from a vertical heated plate 18 in. high at 200 F to still air at 60 F.

Solution:

$$h = 0.3(200 - 60)^{0.25} = 1.03 \text{ B hr}^{-1}\text{ ft}^{-2}\text{ F}^{-1}$$

For free convection between horizontal pipes and still air the following approximate formula is recommended (Ref. VII–1):

$$\left(\frac{\Delta t}{D}\right) \qquad \text{[VII-5]}$$

where D is the external diameter of the pipe in feet units.

Example VII–3. Calculate the heat loss by natural convection from a 4-in. nominal horizontal steam pipe at 200 F to the still air of a room at 70 F.

Solution:

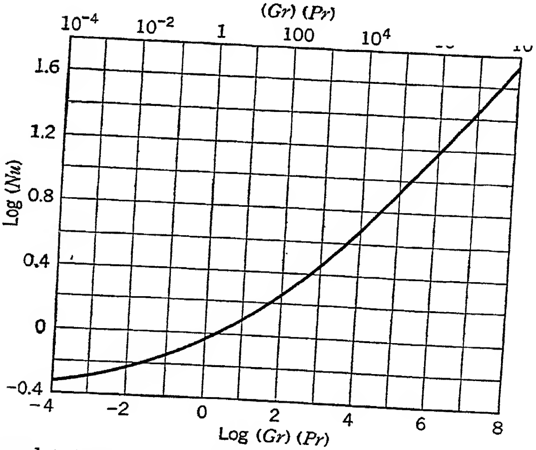
$$h = 0.22\left(\frac{\quad}{4.5/12}\right. \qquad \text{r}^{-1}\text{ ft}^{-}$$

The heat loss per square foot of surface is

$$q'' = h \cdot \Delta t = 0.95(200 - 70) = 123.5 \text{ B hr}^{-1}\text{ ft}^{-2}$$

VII–3 Use of a Dimensionless Correlation

The results of many investigators on the determination of the natural convection coefficients for horizontal cylinders in still gas have been correlated by dimensional analysis. This has shown that for gases of



VII–1. Correlated data for determining coefficients of free convection between horizontal cylinders and diatomic gases.

equal numbers of atoms one can assume $m = n$ in Eqs. VI-1 Thus the latter simplifies to

$$(Nu) = C[(Gr)(Pr)]^n \qquad \text{[VII-6]}$$

The data for diatomic gases are represented by the line drawn in Fig. VII-1 where both coordinates are dimensionless. This chart is very significant in that it represents the correlation of test data for fine horizontal wires in gases as well as for large pipes up to 10 in. in diameter, and temperature differences from cylindrical surfaces to the gas up to about 2000 F for wires and 700 F for the large pipes. For this particular plot, all the fluid properties occurring in Eq. VI-13 are taken at an average temperature between the fluid and the surface and the value of the coefficient of cubical expansion β is assumed to be equal to the reciprocal of the average absolute temperature of the gas, as is true for ideal gases (see Ex. VII-4, footnote).

It is seen from the graph that when $(Gr)(Pr)$ increases from 10^{-4} to 10^{+8}, that is, by 12 decimals, Nu increases only from about 0.5 to 15, that is about 30-fold.

Example VII-4. Calculate the heat loss by natural convection per square foot of outer surface from a horizontal 4-in. nominal steam pipe to the still air of the room if the air and pipe surface temperatures are $t_1 = 70$ F and $t_2 = 200$ F respectively.

Solution: The outer diameter of the pipe $D = 0.375$ ft. The temperature at which the physical properties of the fluid (with the exception of β) are determined is equal to

$$t_m = \frac{200 + 70}{2} = 135 \text{ F}$$

At this temperature the involved physical properties of the air are:

$$\rho = \frac{0.0667}{32.2} = 0.00208 \text{ slug ft}^{-3}$$

$$\mu = 0.1135(10^{-9})3600^2 = 0.00147 \text{ slug ft}^{-1}\text{ hr}^{-1} \text{ (from Table VI-2)}$$

$$c_p = 0.238(32.2) = 7.64 \text{ B slug}^{-1}\text{ F}^{-1}$$

$$k = 0.0161 \text{ B hr}^{-1}\text{ ft}^{-1}\text{ F}^{-1} \text{ (from Fig. II-1)}$$

Further:*

$$\beta = \frac{1}{T_1} = \frac{1}{460 + 70} = 0.00189 \text{ F}^{-1}$$

* In Sect. VI-4 it is mentioned that for ideal gases $\beta = 1/T_1$ where T_1 is the absolute temperature of the main bulk of the gas.

Wherever pounds occur, they are converted to the mass unit slug, by dividing by 32.2. This is necessary because in the derivation of the Grashof number mass units are used (Eq. VI–11, etc.). The usual value of the specific heat is related to 1 lb mass; in order to relate it to 1 slug it must be multiplied by 32.2. (See Sect. IV–1, the paragraphs before Ex. IV–1.)

In addition to the properties of the air, the gravity constant g occurs in (Gr). Because the unit of time used above is the hour, it is necessary to take $g = 32.2(3600)^2$ ft hr^{-2}. Otherwise the consistency of the units in the dimensionless group would be disturbed.

Now, substituting all these magnitudes, one obtains

$$(Gr) \qquad \mu^2 \qquad = 10.75(10^6)$$

$$(Pr) = \frac{\mu c_p}{k} = 0.697, \text{ and}$$

$$(Gr)\ (Pr) = 7.49(10^6)$$

$$\log\ (Gr)\ (Pr) = \log 7.5(10^6) = 6.875$$

Using this number and referring to Fig. VII–1 one finds $\log\ (Nu) = 1.42$ and from this $(Nu) = 26.3$. But $(Nu) = hD/k$. Thus

$$h = \frac{26.3(0.0161)}{0.375} = 1.13 \text{ B hr}^{-1} \text{ ft}$$

and the heat transfer per square foot is equal to

$$q'' = 1.13(200 - 70) = 147 \text{ B hr}^{-1} \text{ ft}^{-2}$$

Referring to the empirical Eqs. VII–1 to 5, it may be mentioned that these too are special cases of Eq. VI–13. In all of them the result, already found by L. Lorenz and mentioned in Sect. VI–4, $m = n = \frac{1}{4}$, is used.

VII–4 The Concept of a Gas Film

The disadvantage of dimensional analysis is that it does not help to explain why such dimensionless groups as the Grashof number occur. A more conspicuous theory is based on the assumption of the existence of a gas film. Langmuir had already visualized the idea of a gas film in his doctor's dissertation in 1906 (Ref. VII–2) and in 1912 he presented this concept in more detail (Ref. VII–3) as follows: Close to a heated wire practically no free convection occurs, and, as a result, a thin layer or film exists through which heat passes by conduction and radiation alone. It is only necessary to consider convection outside of the film.

Langmuir came to this concept through observations which indicated that close to the glowing threads of electric lights convection was very small. This he explained on the basis of the well-known increase of viscosity of gases with the temperature.

Rice (Ref. VII-4) combined this theory with dimensional considerations and arrived at formulas which are rather similar to those found by dimensional analysis alone.

PROBLEMS

VII-1. Calculate the film coefficient of heat transfer by free convection and the total heat transferred from a heated vertical plate 1.5 ft by 2 ft at 200 F to still air at 50 F.

VII-2. Determine the total heat transferred by natural convection from one square foot of a horizontal plate facing upward at a temperature of 300 F to air at 100 F.

VII-3. Calculate the heat transferred per square foot by natural convection from a 6-in. pipe at 200 F in contact with air at 70 F by use of Fig. VII-1.

VII-4. Calculate the heat transfer by natural convection from a horizontal wire 0.05 in. in diameter and 16 in. long at 800 F to nitrogen gas at 100 F.

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CHAPTER VIII

HEAT TRANSFER BY FORCED CONVECTION

VIII–1 Some Peculiarities of Forced Convection

Forced convection is probably the most important method of heat transfer employed in engineering. It is used in almost every type of heat exchanger, at least for one of the fluids and often for both. Forced convection heat transmission also occurs in devices which are not classified as heat exchangers, such as furnaces with artificial draft and thermal engine coolers. It is only at high temperatures that heat transfer by radiation may be more effective.

As in free convection (see Sect. VII–1) the significance of the temperature difference Δt in Eq. I–3 must again be considered. In forced convection the temperature difference is the difference between the well-defined surface temperature t_s and the temperature of the fluid. The latter temperature varies throughout the fluid. In a gas being heated as it flows through a pipe, the fluid temperature is at a maximum value at the surface and at a minimum value at the axis of the pipe. No unique agreement exists as to whether the temperature at the axis, the linear average temperature along a diameter, or another mean temperature should be used. However, the mixing-cup temperature t_m is considered probably the most convenient to use. This could be formed by perfectly mixing the fluid passing the cross-sectional area of the tube in a " cup " and measuring its average temperature. For turbulent flow the temperature of the fluid increases or decreases sharply in a very thin layer next to the surface, and is practically constant throughout the bulk of the fluid; for this the fluid temperature may be taken at any point in the cross-sectional area except close to the surface. The method for determining the average temperature with respect to the temperature change in the direction of the flow of a heated or cooled fluid will be discussed in Sect. IX–6.

For free convection it has been shown by means of dimensional analysis that the Grashof number (Gr) and Prandtl number (Pr) are two groups which may be used as independent variables in setting up empirical equations and constructing graphs, and the heat transfer coefficient may be represented in terms of the dimensionless Nusselt

number (Nu). For forced convection (Nu) may be expressed as a function of Reynolds number (Re) and again the Prandtl group (Pr).

In general, all the dimensional equations which represent the individual coefficient h for forced convection may be expressed by the form

$$(Nu) = C(Re)^m(Pr)^n \qquad \text{[VIII-1]}$$

where C, m, and n are constants.

VIII-2 The Heating of Fluids in Turbulent Flow through Pipes

A good general correlation (Ref. VII-1 and VIII-1) of the data on heating of any fluids in turbulent motion $(Re > 2300)$ through pipes is

$$\cdots p\Big)^{0.4} \qquad \text{[VIII-2]}$$

For cooling of fluids it can be likewise used approximately.

The physical quantities are all determined at the average temperature of the fluid (Ref. VIII-1).

Example VIII-1. Calculate the average film coefficient of heat transfer at the water side of a single-pass steam condenser. The tubes are 0.902 in. inside diameter, and the cooling water enters at 61.4 F and leaves at 69.9 F. The average water velocity is 7 ft/sec.

Solution: h will be found by means of Eq. VIII-2. $D = 0.902/12 = 0.0752$ ft; $v = 7(3600) = 25{,}200$ ft/hr. The pertinent physical properties of water at a mean temperature of 65.7 F, as taken from Tables II-2, VI-1, and from the Keenan and Keyes' Steam Tables (Ref. VIII-2), are:

62.2

$$\mu = 6.0(10^{-9})\text{ lb ft}^{-2}\text{ hr} = 6.0(10^{-9})(3600)^2\text{ slug ft}^{-1}\text{ hr}^{-1}$$

$$= 0.0778\text{ slug ft}^{-1}\text{ hr}^{-1}$$

$$c_p = 1\text{ B lb}_m^{-1}\text{ F}^{-1} = 1(32.2)\text{ B slug}^{-1}\text{ F}^{-1}$$

$$k = 0.341\text{ B hr}^{-1}\text{ ft}^{-1}\text{ F}^{-1}$$

Thus

$$\frac{vD\rho}{\mu} = \frac{\quad}{0.0778} = 47{,}100$$

$$(Pr) = \frac{\mu c_p}{k} = \frac{0.0778(32.2)}{0.341} = 7.34$$

$$(Nu) = \frac{hD}{k} = 0.023(47{,}100)^{0.8}(7.34)^{0.4} = 279.5, \text{ and}$$

$$h = \frac{279.5(0.341)}{0.0752} =$$

In the case of gases the Prandtl number changes very little. It is about 0.67 for monatomic gases, 0.70 for diatomic gases, 0.89 for triatomic gases.

The corresponding values of $(Pr)^{0.4}$ are 0.852, 0.867, and 0.95. Taking approximately $(Pr)^{0.4} = \text{const.} = 0.9$ which corresponds to $(Pr) = 0.794$ the error in (Nu) is in the range of only ± 5 per cent for these three kinds of gases and Eq. VIII–2 simplifies to

$$\frac{hD}{k} = 0.0207 \left(\frac{vD\rho}{\mu}\right)^{0.8}$$

Example VIII–2. If air enters a pipe of 4-in. inside diameter at 65 F and leaves it at 300 F, what is the coefficient of heat transfer? Assume an average air velocity of 15 ft/sec and atmospheric pressure. The viscosity and thermal conductivity values for a mean temperature of 183 F as taken from Table VI–2 and Fig. II–1 are $0.119(10^{-9})$ lb ft^{-2} hr and 0.0171 B hr^{-1} ft^{-1} F^{-1} respectively. The specific weight is 0.0617 lb ft^{-3}.

Solution: With $D = 4/12$ ft, $v = 15(3600)$ ft/hr, $\rho = 0.0617/32.2$ slug/ft^3 and $\mu = 0.119(10^{-9})3600^2$ slug ft^{-1} hr^{-1}, one obtains

$$(Re) = \frac{vD\rho}{\mu} = (15)3600(\tfrac{4}{12})\,\frac{0.0617}{32.2}\cdot\frac{10^9}{0.119(3600^2)} = 22{,}400$$

By substitution in Eq. VIII–3

$$Nu = \frac{h(\frac{4}{12})}{0.0171}$$

and $h = 3.23$ B hr^{-1} ft^{-2} F^{-1}.

VIII–3 The Heating of Liquids in Streamline Flow Through Pipes

The equations which are used for streamline flow in pipes with viscous liquids like oils are more complicated than those for turbulent flow.

Sieder and Tate (Ref. VIII–1 and 3) in experiments with viscous petroleum oils in tubes with inner diameters ranging from 0.39 to 1.57 in. and heated lengths ranging from 3 to 11.6 ft, arrived at the following formula:

$$\frac{hD}{k} = 1.86\left(\frac{vD\rho}{\mu}\right)^{1/3}\left(\frac{\mu c_p}{k}\right)^{1/3}\left(\frac{D}{L}\right)^{1/3}\left(\frac{\mu_m}{\mu_s}\right)^{0.14}$$

or

$$(Nu) = 1.86(Re)^{1/3}(Pr)^{1/3}\left(\frac{D}{L}\right)^{1/3}\left(\frac{\mu_m}{\mu_s}\right)^{0.14} \qquad \text{[VIII-4]}$$

where h is related to the difference between the mean surface temperature t_s and the arithmetic mean t_m of the " cup " temperature of the liquid at entrance and exit;

L is the heated length of the tube;

μ_m is the dynamic viscosity of the liquid at temperature t_m; and

μ_s is the same at mean surface temperature t_s.

In the experiments μ_m/μ_s ranged from 0.004 to 9.8.

All factors in Eq. VIII–4 are dimensionless. Comparison with Eq. VIII–1 however shows that there are two more dimensionless terms D/L and μ_m/μ_s involved. Geometrical similarity of tubes obviously exists only if D/L is a constant. This is the reason why D/L appears in the equation as a parameter. In turbulent flow, the exponent of D/L is about $1/18$ and $(D/L)^{1/18}$ approaches a practically constant value if L is relatively great compared with D. Thus, in turbulent flow in relatively long tubes Eq. VIII–2 can be used without taking care of the small variability with D/L in that range. This is not true with streamline flow where, according to experience, the exponent is only $1/3$. The dimensionless ratio μ_m/μ_s considers the influence of temperature on viscosity which already has been mentioned in Sect. VI–5. By means of this term Eq. VIII–4 can be used for heating and for cooling of viscous oils in tubes, whereas without this term different equations for these two cases would be necessary.

VIII–4 The Heating of Fluids Flowing Normal to Single Wires and Tubes

For this case, as for gas flow inside of a tube, one can use the equation

$$(Nu) = C(Re)^m \qquad \text{[VIII–5]}$$

Different investigators found rather different values for C and m. As an average $C = 0.3$ and $m = 0.57$ may be assumed.

Example VIII–3. Calculate the heat transfer coefficient for the case of air flowing normal to a 1-in. pipe at a Reynolds number of 8000 if the average surface temperature of the pipe is 183 F.

Solution: $D = 1/12$ ft, and $k = 0.0171$ B hr^{-1} ft^{-1} F^{-1} (see Ex. VIII–2). By substitution of $(Re) = 8000$ in Eq. VIII–5

$$0.3(8000)^{0.57} = 50.0$$

and

$$h = \frac{50.0(0.0171)}{1/12} = 10.3 \text{ B hr}^{-1}\text{ ft}^{-2}$$

Since the constants of Eq. VIII–5 are not known exactly, it does not introduce an appreciable error if k is taken at the surface temperature instead of at an average between the gas and surface temperatures.

For the heat transfer from tubes to liquids, Eq. VIII–1 may be used with $C = 0.6$, $m = 0.5$, and $n = 0.31$ in the range from $Re = 50$ to $Re = 10{,}000$ (Ref. VIII–4).

VIII–5 The Heating of Fluids Flowing Normal to Banks of Staggered Tubes

Here Colburn (Ref. VIII–5) recommends the following relation for correlating the variables which influence the heat transfer coefficient:

$$\left(\frac{h}{v_m \rho c_p}\right)\left(\frac{\mu c_p}{k}\right)^{2/3} = 0.33\left(\frac{v_m D \rho}{\mu}\right)^{-0.4} \qquad \text{[VIII-6]}$$

In this formula D is the outer diameter of the tubes, and v_m is the mean velocity in the smallest free cross-sectional area between the tubes passed by the fluid. The viscosity and thermal conductivity values are determined at the average film temperature.

$$t_f = t_m + \frac{t_s - t_m}{2} \qquad \text{[VIII-7]}$$

where t_m and t_s are the mean fluid and surface temperatures respectively.

Equation VIII–6 can be converted to

$$\frac{hD}{k} = 0.33\left(\frac{v_m D \rho}{\mu}\right)^{0.6}\left(\frac{\mu c_p}{k}\right)^{1/3}$$

or $\qquad$ [VIII-8]

$$(Nu) = 0.33(Re_m)^{0.6}(Pr)^{1/3}$$

as can be easily checked (Ref. VIII–1).

VIII–6 The Surface Film Theory of Heat Transfer

The equations appearing in this chapter are based on dimensional analysis. However, as for free convection, there exists a surface film theory based on more prominent facts.

In 1904 L. Prandtl first directed attention to the different behavior of the bulk of a fluid in turbulent motion and of a thin layer on the solid walls which limits the flow. In the former mixing movements prevail, whereas in the latter gas friction is characteristic for the motion. In 1910 Prandtl showed in an analogous manner that heat is transferred in the bulk of the flow by mixing movements only, and in a surface film by thermal conduction alone. Using Reynolds' idea that gas friction and heat transfer are governed by the same laws (see Chapter XV),

Prandtl developed a theory concerning this analogy which is known as the surface film theory of forced heat convection. This theory is still in a state of development.

As a result of Langmuir's concept of a gas film, which incidentally is very similar to Prandtl's assumption, Rice developed formulas for forced convection (Ref. VIII–6) which, like those for free convection, are not very different from the relations found by dimensional analysis.

PROBLEMS

VIII–1. A cylinder 1 in. in diameter is placed in a hot air duct at right angles to the direction of the air flow. If the air and cylinder temperatures are 100 F and 300 F respectively, calculate the value of the film coefficient for a mass flow velocity of 8000 $lb_m\ hr^{-1}\ ft^{-2}$ (see Sect. VI–2).

VIII–2. Water at a rate of 5000 lb/hr flows through a 2-in. internal-diameter tube. The temperatures of the water entering and leaving are 80 F and 120 F respectively. Calculate the heat transfer coefficient at the water side.

VIII–3. Air flows through a tube having an internal diameter of 4 in. at a volume rate of 10,000 ft^3/hr at 130 F. If the entrance and discharge temperatures are 60 F and 200 F respectively, calculate the heat transfer coefficient. Assume an average air pressure of 40 lb/sq in. absolute.

VIII–4. Determine the coefficient of heat transfer for oil flowing through a heated tube 8 ft long and 1 in. in diameter at an average velocity of 1.5 ft/sec. The entering and discharge oil temperatures are 129.1 F and 130.9 F respectively. Assume that the tube surface temperature is constant and equal to 160 F. The specific gravity,* average thermal conductivity, and average specific heat values for the oil are 0.88, 0.08 $B\ hr^{-1}\ ft^{-1}\ F^{-1}$, and 0.5 $B\ lb_m^{-1}\ F^{-1}$ respectively. The variation of the dynamic viscosity of the oil for various temperatures may be obtained from the following data:

Temperature F	Dynamic Viscosity 10^{-3} lb ft^{-2} sec
120	1.3
130	1.1
140	0.9
150	0.7
160	0.6

VIII–5. Calculate the heat transfer coefficient for water flowing over a bank of staggered tubes 1 in. in diameter at a mean velocity through the smallest free area of 8000 ft/hr. Assume that the average film temperature is 118 F.

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* Title translated by the authors.

CHAPTER IX

HEAT TRANSFER BY THE COMBINED EFFECT OF CONDUCTION AND CONVECTION

IX-1 Cases of Combined Conduction and Convection

Thus far only such cases have been treated where either conduction and convection occurs alone or the effect of the one prevails so much that the other can be neglected. This is not generally true.

There are two main cases in which the combined effect of conduction and convection must be considered. The one deals with the projections of a surface into the environment, as for instance, the handle of a hot vessel or fins on the cylinder of a combustion engine. Here heat is conducted from the root to the free end or edge of the projection, and at the same time heat is given up to the environment by convection and by radiation. The radiation effect will be neglected or simply considered as included in the convection effect in this chapter.

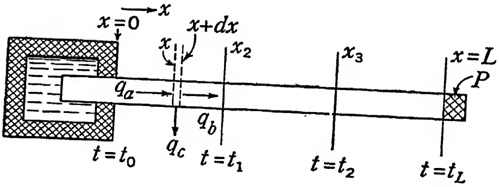
The second case occurs at the wall of a building and in heat exchangers. In a building, heat energy may be transferred from the inside air to the wall, through the wall by conduction, and finally given up to the outside air. Thus, there is a series of one convective and one conductive and another convective transfer of the same rate of heat, similar to the heat flow by conduction through a series of different layers of insulation of a composite wall which has been dealt with in Sect. III-3 and 6. Consider the transfer of heat from the hot water to the cold water in a tubular water cooler. The temperature of the hot water is reduced as it passes through the apparatus owing to the gradual dissipation of thermal energy of the hot water particles to the cooler surface of the walls which separate the hot and cold water. The thermal energy then flows through the metal wall by conduction and finally into the cold water by convection.

IX-2 Heat Transfer from a Rod Heated on One End

The first of these cases will be treated by using as an example the heat transfer from a rod of arbitrary constant cross section protruding from a vessel at constant temperature.

In Fig. IX-1 a rod whose cross section has an area A and a circumference C protrudes by the length $x = L$ from a perfectly heat-insulated

liquid bath at constant temperature t_0 into a room in which the air temperature is t_a. The rod is made of a material having a thermal conductivity of k, and the film coefficient of heat transfer from the rod surface to the air may be h, both k and h being assumed the range of temperatures considered. It is



IX–1. Rod protruding from a liquid bath.

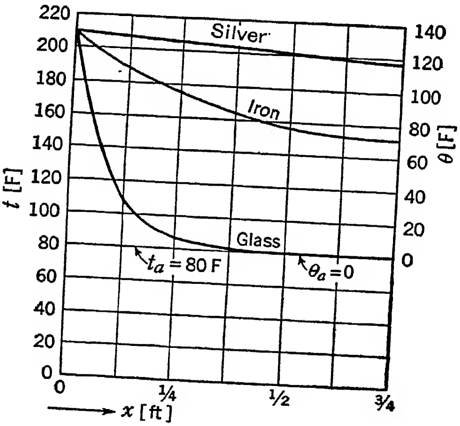


FIG. IX–2. Temperature distribution along the rod, shown in Fig. IX–1.

the heat balance for
s perpendicular to the
body at constant temperature t_0. There is a
y conduction q_a, originating from the liquid bath, and another one q_b in the direction of the free end of the rod. Furthermore, an amount q_c is given up by convection from the rod to the air. Under steady state conditions, there will be a heat balance of incoming and outgoing heat energy

the at

according to Eq. III–1

$$q_a = -kA\frac{dt}{dx} \tag{IX-2}$$

Furthermore, according to Eq. IV–5

$$q_b = q_a - kA\frac{d^2t}{dx^2}dx \tag{IX-3}$$

Finally, according to Eq. I–3

$$q_c = h(C \cdot dx)\,(t - t_a) \tag{IX-4}$$

because the surface of the differential slice through which heat is given up to the environment is $C \cdot dx$, and the difference between its temperature and that of the air is $t - t_a$.

Substituting q_b and q_c from Eqs. IX–3 and 4 in Eq. IX–1 one obtains

$$q_a = q_a - kA\frac{d^2t}{dx^2}dx + hC \cdot dx(t - t_a)$$

or

$$\frac{d^2t}{dx^2} = \frac{hC}{kA}(t - t_a) \tag{IX-5}$$

Let $\theta = (t - t_a)$ be the temperature of the rod, referred to the air temperature as zero, and simplify by introducing the symbol $m = \sqrt{hC/kA}$.

Then, because $t_a = \text{const.}$, $d^2t/dx^2 = d^2\theta/dx^2$. Thus Eq. IX–5 simplifies to

$$\frac{d^2\theta}{dx^2} = m^2\theta \tag{IX-6}$$

The general solution of this differential equation is

$$\theta = Me^{-mx} + Ne^{mx} \tag{IX-7}$$

where M and N are two arbitrary constants of integration. It is easy to check Eq. IX–7. Differentiating once gives

$$\frac{d\theta}{dx} = -Mme^{-mx} + Nme^{mx} \tag{IX-8}$$

Differentiating a second time gives:

$$\frac{d^2\theta}{dx^2} = Mm^2e^{-mx} + Nm^2e^{mx} = m^2(Me^{-mx} + Ne^{mx})$$

and by substitution from Eq. IX–7

$$\frac{d^2\theta}{dx^2} = m^2\theta \qquad \text{[IX-6]}$$

The constants M and N will be found from the so-called boundary conditions, two of them being necessary and sufficient.

If, for instance, the free end of the rod in Fig. IX–1 were perfectly insulated by a pad P, then q_a would be zero at $x = L$, that is, no heat could leave the rod in the axial direction at $x = L$. Thus according to Eq. IX–2

$$(q_a)_{x=L} = -kA\left(\frac{dt}{dx}\right)_{x=L} = 0$$

or

$$\left(\frac{dt}{dx}\right)_{x=L} = \left(\frac{d\theta}{dx}\right)_{x=L} = 0$$

Substituting this and $x = L$ in Eq. IX–8 gives a first boundary condition,

$$0 = -Mme^{-mL} + Nme^{mL}$$

or

$$e^{2mL} = \frac{M}{N} \qquad \text{[IX-9]}$$

A second boundary condition is found from the fact that owing to the perfect insulation of the liquid bath, at $x = 0$ the rod temperature is $t = t_0$, or $\theta = \theta_0$. Substituting this in Eq. IX–7 gives

$$\theta_0 = Me^0 + Ne^0 = M + N \qquad \text{[IX-10]}$$

From Eqs. IX–9 and 10 it follows

$$M = \frac{\theta_0}{1 + e^{-2mL}} \quad \text{and} \quad N = \frac{\theta_0}{1 + e^{2mL}} \qquad \text{[IX-11]}$$

and by substitution in Eq. IX–7

$$\theta = \theta_0\left(\frac{e^{-mx}}{1 + e^{-2mL}} + \frac{e^{mx}}{1 + e^{2mL}}\right) \qquad \text{[IX-12]}$$

Example IX–1. A cylindrical rod of ¾-in. diameter and 9 in. long protrudes from a well-insulated steam vessel at 210 F into air of 80 F. The free end of the rod is insulated so that the heat losses from this end may be negligible. Determine the temperature at the free end if the rod is made of (a) silver, (b) iron, and (c) glass. Show the temperature distribution along the rod in these three cases. Take the heat conductivities of the three substances equal to 256, 36, and 0.64 B hr^{-1} ft^{-1} F^{-1} respectively, and in all three cases assume $h = 1.44$ B hr^{-1} ft^{-2} F^{-1}.

Solution: Substituting $x = L$ in Eq. IX-12 yields the temperature at the end of the rod

$$\theta_L = \theta_0\left(\frac{e^{-mL}}{1 + e^{-2mL}} + \frac{e^{mL}}{1 + e^{2mL}}\right)$$

This can be simplified by reducing to a common denominator:

$$\theta_L = \theta_0 \frac{e^{-mL}(1 + e^{2mL}) + e^{mL}(1 + e^{-2mL})}{(1 + e^{-2mL})\,(1 + e^{2mL})}$$

$$= \theta_0 \frac{2(e^{mL} + e^{-mL})}{e^{2mL} + 2 + e^{-2mL}} = \theta_0 \frac{2}{e^{mL} + e^{-mL}} \qquad \text{[IX-13]}$$

$$\theta_0 = t_0 - t_a = 130\ \text{F}$$

$$L = 9/12 = 0.75\ \text{ft}$$

$$m = \sqrt{hC/kA}$$

$$h = 1.44\ \text{B hr}^{-1}\ \text{ft}^{-2}\ \text{F}^{-1}$$

$$C = \pi \frac{3/4}{12}\ \text{ft}$$

$$A = \frac{\pi}{4}\left(\frac{3/4}{12}\right)^2\ \text{ft}^2$$

$$\frac{C}{A} = \frac{4}{\dfrac{3/4}{12}} = 64\ \text{ft}^{-1}$$

It is necessary to convert all inch units into foot units since foot is used as a unit length in h and k. It may be shown that the dimension of m is ft^{-1} so that the exponents mx and mL in the above equations become dimensionless. As mentioned in Sect. VII-2 the exponents of a physically and mathematically exact equation must be dimensionless.

With the above values

$$m = \sqrt{\frac{1.44(64)}{k}} = 9.6\frac{1}{\sqrt{k}}\ \text{ft}^{-1}$$

and by substitution in Eq. IX-13 for case

(*a*) $k = 256$ $\qquad m = \dfrac{9.6}{\sqrt{256}} = \dfrac{9.6}{16} = 0.6\ \text{ft}^{-1}$ $\qquad mL = 0.45$

(*b*) $k = 36$ $\qquad m = \dfrac{9.6}{6} = 1.6\ \text{ft}^{-1}$ $\qquad mL = 1.2$

(*c*) $k = 0.64$ $\qquad m = \dfrac{9.6}{0.8} = 12\ \text{ft}^{-1}$ $\qquad mL = 9$

Furthermore:

(a) $e^{mL} = e^{0.45} = 1.568$

(b) $e^{mL} = e^{1.2} = 3.285$

(c) $e^{mL} = e^{9} = 8104$

and by substitution in Eq. IX–12

(a) $$\theta_L = 130 \frac{2}{1.568 + \dfrac{1}{1.568}} = 118 \text{ F and } t_L = \theta_L + t_a = 118 + 80 = 198 \text{ F}$$

(b) $$\theta_L = 130 \frac{2}{3.285 + \dfrac{1}{3.285}} = 72.5 \text{ F and } t_L = 72.5 + 80 = 152.5 \text{ F}$$

(c) $$\theta_L = 130 \frac{2}{8104 + \dfrac{1}{8104}} = 0.032 \text{ F and } t_L = 0.03 + 80 = 80.03 \text{ F}$$

From these computations it is seen that the temperature of the free end of the silver rod is only 12 F below that of the heated end, that of the iron rod is almost at the mean between the steam and air temperatures, and that of the glass rod is practically equal to the air temperature.

The temperature distribution in the rods may be calculated by substituting different values for x in Eq. IX–12.

The calculation will be performed for $x_1 = 0.25$ ft and $x_2 = 0.5$ ft.

(a) $e^{m(0.25)} = e^{0.15} = 1.162$

$e^{m(0.5)} = e^{0.3} = 1.350$

$e^{2mL} = e^{m(1.5)} = e^{0.9} = 2.460$

$$\theta_1 = 130\left(\frac{\dfrac{1}{1.162}}{1 + \dfrac{1}{2.460}} + \frac{1.162}{1 + 2.460}\right) = 130\left(\frac{0.860}{1.407} + \frac{1.162}{3.460}\right) = 123.2 \text{ F}$$

$$t_1 = 203.2 \text{ F}$$

$$\theta_2 = 130\left(\frac{\dfrac{1}{1.350}}{1.407} + \frac{1.350}{3.460}\right) = 130(0.527 + 0.390) = 119.2 \text{ F}$$

$$t_2 = 199.2 \text{ F}$$

(*b*) $e^{m(0.25)} = e^{0.4} = 1.492$

$e^{m(0.5)} = e^{0.8} = 2.226$

$e^{2mL} = e^{2.4} = 11.02$

$$\theta_1 = 130\left(\frac{\frac{1}{1.492}}{1+\frac{1}{11.02}} + \frac{1.492}{1+11.02}\right) = 130\left(\frac{0.671}{1.091} + \frac{1.492}{12.02}\right) = 96.2\text{ F}$$

$$t_1 = 176.2\text{ F}$$

$$\theta_2 = 130\left(\frac{\frac{1}{2.226}}{1.091} + \frac{2.226}{12.02}\right) = 130(0.412+0.185) = 77.5\text{ F}$$

$$t_2 = 157.5\text{ F}$$

(*c*) $e^{m(0.25)} = e^{3} = 20.09$

$e^{m(0.5)} = e^{6} = 403.5$

$e^{2mL} = e^{18} = 65{,}700{,}000$

$$\theta_1 = 130\left(\frac{\frac{1}{20.09}}{1+0} + \frac{20.09}{65{,}700{,}000}\right) = 130(0.0498 + 0) = 6.5\text{ F}$$

$$t_1 = 86.5\text{ F}$$

$$\theta_2 = 130\left(\frac{\frac{1}{403.5}}{1} + \frac{403.5}{65{,}700{,}000}\right) = 130(0.00248 + 0) = 0.32\text{ F}$$

$$t_2 = 80.3\text{ F}$$

The temperature distributions, according to these figures, are shown in Fig. IX–2. All the curves become tangent to the horizontal axis, which is true only for a perfectly insulated pad P as shown in Fig. IX–1.

If the end of the rod was uninsulated, then with the value of h at $x = L$, according to Eq. IX–2

$$(q_a)_{x=L} = -kA\left(\frac{dt}{dx}\right)_{x=L} = -kA\left(\frac{d\theta}{dx}\right)_{x=L}$$

as before. However, for this case it is not equal to zero, and according to Eq. I–3

$$(q_a)_{x=L} = hA\,(t_{x=L} - t_a) = hA\theta_{x=L}$$

Equating both expressions one obtains

$$-k\left(\frac{d\theta}{dx}\right)_{x=L} = h\theta_{x=L} \qquad \text{[IX–14]}$$

But from Eq. IX–8 it follows that

$$\left(\frac{d\theta}{dx}\right)_{x=L} = -Mme^{-mL} + Nme^{mL}$$

and from Eq. IX–7

$$\theta_{x=L} = Me^{-mL} + Ne^{mL}$$

By substitution in Eq. IX–14 the first boundary condition is obtained

$$km(Me^{-mL} - Ne^{mL}) = h(Me^{-mL} + Ne^{mL}) \qquad \text{[IX–15]}$$

and the second boundary condition is given by Eq. IX–10 as before. Equations IX–10 and 15 may be solved for the two unknown quantities M and N, and the resulting expressions are substituted in Eq. IX–7.

Thus, the procedure is exactly the same as described before; only the calculation is more cumbersome.

The total rate of heat given up by a rod is found by considering that all this heat must cross the area A at $x = 0$. Therefore it is necessary to compute

$$q_{x=0} = -kA\left(\frac{dt}{dx}\right)_{x=0} = -kA\left(\frac{d\theta}{dx}\right)_{x=0} \qquad \text{[IX–16]}$$

or, by substitution from Eq. IX–8, using the denotation q_0 instead of $q_{x=0}$

$$q_0 = kAm(M - N) \qquad \text{[IX–17]}$$

EXAMPLE IX–2. Calculate the rate of heat given up by the iron rod in Ex. IX–1.

Solution: Substituting M and N from Eq. IX–11 gives

$$M - N = \theta_0\left(\frac{1}{1 + e^{-2mL}} - \frac{1}{1 + e^{2mL}}\right)$$

This can be simplified by employing a common denominator. Then

$$M - N = \theta_0 \frac{e^{2mL} - e^{-2mL}}{1 + e^{2mL} + e^{-2mL} + 1} = \theta_{0m} \frac{(e^{mL} + e^{-mL})\,(e^{mL} - e^{-mL})}{(e^{mL} + e^{-mL})^2}$$

$$= \theta_0 \frac{e^{mL} - e^{-mL}}{e^{mL} + e^{-mL}} = \theta_0 \tanh(mL) \qquad \text{[IX–18]}$$

and by substitution in Eq. IX–17:

$$q_0 = kAm\theta_0 \frac{e^{mL} - e^{-mL}}{e^{mL} + e^{-mL}} = kAm\theta_0 \tanh(mL) \qquad \text{[IX–19]}$$

Substituting the data given for the iron rod:

$$q_0 = 36\left(\frac{\pi}{4}\,\frac{9/16}{144}\right) 1.6\ (130)\tanh 1.2 = 22.96\tanh 1.2 = 22.96(0.834)$$
$$= 19.1 \approx 19\ \mathrm{B/hr}$$

Finally, it may be mentioned that the above equations are not restricted to cylindric rods, but hold likewise for any projections from a heating surface, for instance, fins on a plane plate where A represents the area of the cross section through the fin, and C the circumference of this area.

IX–3 Heat Transmission Between Two Fluids Through a Plane Wall

The combined effect of conduction and convection in heat exchangers may be considered in a manner similar to heat conduction through a composite wall as shown in Sect. III–3.

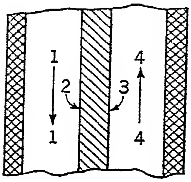


Fig. IX–3. Counterflow through channels with plane separating walls.

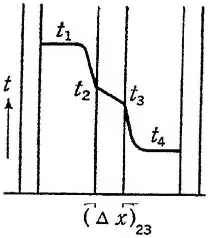


Fig. IX–4. Temperature distribution in the system of Fig. IX–3.

Figure IX–3 represents a cross section through two channels 1 and 4 with a common plane separating wall, the surfaces of which may be 2 and 3. Fluids at different temperatures are assumed to flow in the two channels, for example, in opposite direction (counter-flow) as indicated by the two arrows. For the present, the length of the heat exchanger is considered such that the temperature of each fluid may be assumed to be constant. Let the temperatures of the fluids be t_1 and t_4 (see Fig. IX–4), and the surface temperatures of the separating wall t_2 and t_3, its thermal conductivity k_{23} and the film coefficients of heat transfer h_{12} and h_{34} respectively. Similarly as in heat conduction through a com-

posite wall, the rate of heat flowing perpendicularly to the surfaces and 2 through an area A is

$$q = h_{12}A\,(t_1 - t_2) = k_{23}A\,\frac{t_2 - t_3}{(\Delta x)_{23}} = h_{34}A\,(t_3 - t_4)$$

From this:

$$t_1 - t_2 = \frac{q}{h_{12}A}$$

$$t_2 - t_3 = \frac{q(\Delta x)_{23}}{k_{23}A}$$

$$t_3 - t_4 = \frac{q}{h_{34}A}$$

By adding these three temperature differences it follows that

$$t_1 - t_4 = \frac{q}{A}\left[\frac{1}{h_{12}} + \frac{(\Delta x)_{23}}{k_{23}} + \frac{1}{h_{34}}\right]$$

or

$$q = \frac{A\,(t_1 - t_4)}{\dfrac{1}{h_{12}} + \dfrac{(\Delta x)_{23}}{k_{23}} + \dfrac{1}{h_{34}}} = UA\,(t_1 - t_4) \qquad \text{[IX-20]}$$

which corresponds to Eq. III-3. For simplification a new symbol

$$U = \frac{1}{\dfrac{1}{h_{12}} + \dfrac{(\Delta x)_{23}}{k_{23}} + \dfrac{1}{h_{34}}} \qquad \text{[IX-21]}$$

has been introduced. The magnitude U is called the overall coefficient of heat transfer. Its physical dimensions are the same as those of the film coefficient of heat transfer.

Since in most practical cases the temperatures t_1 and t_4 of the two fluids are known or may be measured or calculated easily, whereas the measurement of the surface temperatures t_2 and t_3 is difficult or even impossible, the overall coefficient has considerable practical importance.

Example IX-3. A heat exchanger wall consists of a copper plate ⅜ in. thick. If the two surface coefficients are 480 and 1250 B hr^{-1} ft^{-2} F^{-1} respectively, calculate the overall heat transfer coefficient. Assume that k for the given problem is 220 B hr^{-1} ft^{-1} F^{-1}.

Solution:

$$U = \frac{1}{\dfrac{1}{480} + \dfrac{0.375/12}{220} + \dfrac{1}{1250}} = \frac{1}{0.00302} = 331 \text{ B hr}^{-1}\text{ ft}^{-2}\text{ F}^{-1}$$

As in heat conduction through a composite wall (Sect. III-4) heat ances may be used. Equation IX-20 then assumes the simple form

$$q = \frac{t_1 - t_4}{R_{12} + R_{23} +} \qquad \text{[IX-20}a\text{]}$$

which corresponds to Eq. III-3*a*.

IX-4 Heat Transmission Between Two Fluids Through a Cylindric Wall

In the case of a double tube heat exchanger, as represented in Fig. IX-5, the procedure is similar to that with thermal conduction through several layers of insulation on a cylinder (see Sect. III-6).

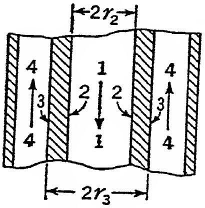


FIG. IX-5. Double tube heat exchanger.

The nomenclature is analogous to that used for Figs. IX-3 and 4.

The flow of heat per unit length of the concentric tubes of Fig. IX-5 may be represented by

$$q' = h_{12} \cdot 2\pi r_2(t_1 - t_2) = \frac{k_{23} \cdot 2\pi(t_2 - t_3)}{\ln r_3/r_2}$$
$$= h_{34} \cdot 2\pi r_3(t_3 - t_4)$$

or

$$t_1 - t_2 = \frac{q'}{h_{12}2\pi r_2}$$

$$t_2 - t_3 = \frac{q'}{\dfrac{k_{23}2\pi}{\ln r_3/r_2}}$$

$$t_3 - t_4 = \frac{q'}{h_{34}2\pi r_3}$$

By addition

$$t_1 - t_4 = q'\left(\frac{1}{h_{12}2\pi r_2} + \frac{\ln r_3/r_2}{k_{23}2\pi} + \frac{1}{h_{34}2\pi r_3}\right)$$

or

$$q' = \frac{t_1 - t_4}{\dfrac{1}{h_{12}2\pi r_2} + \dfrac{\ln r_3/r_2}{k_{23}2\pi} + \dfrac{1}{h_{34}2\pi r_3}} \quad \text{[IX-22]}$$

Referring to a length L of the double pipe let

$$q = q'L \quad \text{[IX-23]}$$

$$A_2 = 2\pi r_2 \cdot L \quad \text{[IX-24]}$$

$$A_3 = 2\pi r_3 \cdot L \quad \text{[IX-25]}$$

From Eqs. IX–22 to 25 the following is obtained:

$$q = U_2A_2(t_1 - t_4) = U_3A_3(t_1 - t_4) \quad \text{[IX-26]}$$

Here U_2 and U_3 are overall coefficients of heat transfer, which are related to inner and outer areas A_2 and A_3, respectively, by the equations

$$U_2 = \frac{1}{\dfrac{1}{h_{12}} + \dfrac{A_2 \ln r_3/r_2}{k_{23}2\pi L} + \dfrac{A_2}{h_{34}A_3}} \quad \text{[IX-27]}$$

and

$$U_3 = \frac{1}{\dfrac{A_3}{h_{12}A_2} + \dfrac{A_3 \ln r_3/r_2}{k_{23}2\pi L} + \dfrac{1}{h_{34}}} \quad \text{[IX-28]}$$

These may easily be checked.

Furthermore, it may be considered that, according to Eq. IX–25, $A_3/L = 2\pi r_3$. Therefore the second term in the denominator of Eq. IX–28 equals $\dfrac{r_3 \ln r_3/r_2}{k_{23}}$. Referring to the first term, it is seen that $\dfrac{A_3}{A_2} = \dfrac{2\pi r_3 \cdot L}{2\pi r_2 \cdot L} = \dfrac{r_3}{r_2}$. So this term becomes $\dfrac{r_3}{h_{12}r_2}$, and

$$U_3 = \frac{1}{\dfrac{r_3}{h_{12}r_2} + \dfrac{r_3 \ln r_3/r_2}{k_{23}} + \dfrac{1}{h_{34}}} \quad \text{[IX-28a]}$$

Example IX–4. Calculate the overall heat transfer coefficient based on the outer area for a copper condenser tube of ¾-in. outside diameter having a wall thickness of 0.1 in. Assume that the inner film coefficient is 280 B hr^{-1} ft^{-2} F^{-1}, the outer film coefficient is 2000 B hr^{-1} ft^{-2} F^{-1}, and the thermal conductivity of copper is 200 B hr^{-1} ft^{-1} F^{-1}.

Solution: From Eq. IX–28*a*:

$$U_3 = \frac{1}{0.00488 + 0.0000485 + 0.0005} = \frac{1}{0.00543} = 184 \text{ B hr}^{-1}\text{ ft}^{-2}\text{ F}^{-1}$$

IX–5 Types of Heat Exchangers

Thus far the length L of a heat exchanger was considered such that the temperatures of the two fluids could be assumed constant throughout this distance. Generally, the temperatures change in the direction of the flow and the question arises as to what average value of $t_1 - t_4$ will take into account the temperature variations.

In order to consider the mean temperature difference, the temperature distribution in the various types of heat exchangers will be considered. The subscripts a and b refer to the two ends of the exchanger and the prime sign to the heating fluid.

All heat exchangers may be divided into the following five general classes.

Class 1. Heat exchangers wherein a fluid at a constant temperature gives up heat to a colder fluid the temperature of which gradually

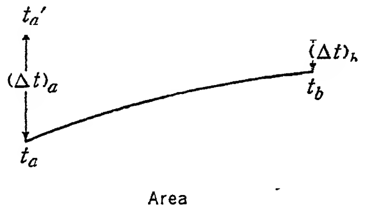


Fig. IX–6. Temperature distribution in heat exchanger of class 1 (condenser).

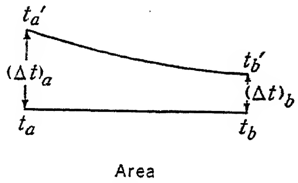


Fig. IX–7. Temperature distribution in heat exchanger of class 2 (boiler).

increases as it flows through the device. The heating fluid can be at rest or moving in any direction. An example of this type would be a steam condenser. Figure IX–6 indicates the temperature distribution in the apparatus.

Class 2. Devices wherein a fluid at constant temperature receives heat from a warmer fluid the temperature of which decreases as it flows through the exchanger. Here the heated fluid can be at rest or moving in any direction. A steam boiler serves as an example of this class. Figure IX–7 shows the temperature distribution in this type of heat exchange apparatus.

Class 3. Parallel flow heat exchangers wherein the fluids flow in the same direction and both of them change their temperature. Many devices such as water heaters and oil heaters and coolers fall in this group. The temperature distribution for this type of apparatus is illustrated in Fig. IX–8.

Class 4. Counter-flow heat exchangers wherein the fluids flow in directions opposite to one another. This possibly is the most favorable

kind of fluid heaters and coolers. A temperature distribution diagra for this type of apparatus is shown in Fig. IX-9.

Class 5. Cross-flow heat exchangers in which one fluid flows at a angle to the second one, as is the case in tube banks. This group w not be dealt with in the present chapter.

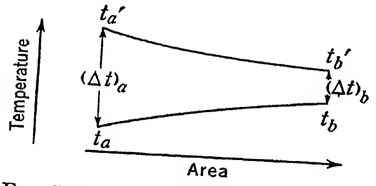


FIG. IX–8. Temperature distribution in heat exchanger of class 3 (parallel flow).

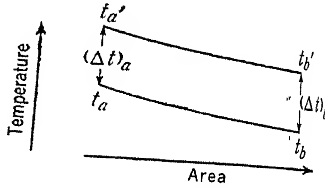


FIG. IX–9. Temperature distribution in heat exchanger of class 4 (counter-flow).

IX–6 The Log Mean Temperature Difference

A relation for the mean temperature diff holds for heat exchangers of any of the

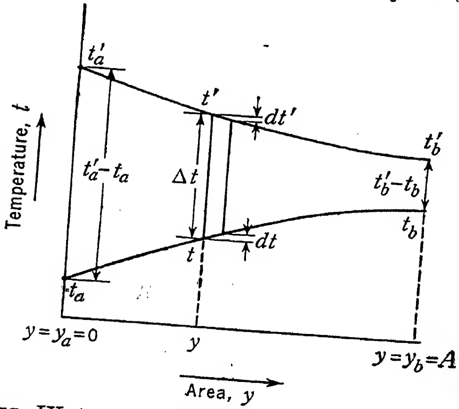


FIG. IX–10. Temperature distribution diagram for a parallel-flow heat exchanger.

IX–10 which constitutes a schematic temperature distribution diagram for a parallel-flow heat exchanger.

The rate of heat transferred through area dy located distance x from the entrance of the heat exchanger is

$$dq = U \cdot dy(t' - t) \quad \text{[IX–29]}$$

The rates of heat dq_h lost by the hot fluid and dq_c gained by the cold fluid in passing over area dy are equal to the product of the rate of

specific heat c_h (or c_c), and temperature change dt' Thus

$$-m_h c_h dt' \quad \text{[IX–30]}$$

$$+m_c c_c dt \quad \text{[IX–31]}$$

Combining these two equations and dropping the subscripts of q since the heat gained by the cold fluid is equal to the heat lost by the hot fluid, gives

$$dt' - dt = -dq\left(\frac{1}{m_h c_h} + \frac{1}{m_c c_c}\right)$$

Substituting $d(t' - t)$ for $(dt' - dt)$ and a new symbol N defined by

$$N = \frac{1}{m_h c_h} + \frac{1}{m_c c_c} \qquad \text{[IX-32]}$$

the following is obtained:

$$d(t' - t) = -dq \cdot N$$

Substituting for dq from Eq. IX-29

$$\frac{d(t' - t)}{(t' - t)} = -UN \cdot dy$$

By integrating

$$\ln\,(t' - t) = -UN\,y + C \qquad \text{[IX-33]}$$

One boundary condition is

$$t' - t = t_a' - t_a$$

for $y = 0$, that is, at the flow entrance. Substituting this in Eq. IX-33 yields

$$C = \ln\,(t_a' - t_a)$$

and again substituting in Eq. IX-33 gives

$$\ln\frac{t' - t}{t_a' - t_a} = -UN\,y \qquad \text{[IX-34]}$$

Another boundary condition is

$$t' - t = t_b' - t_b$$

for $y = A$, that is, for the whole heating area of the heat exchanger. Substituting this in Eq. IX-34 leads to

$$\frac{1}{N}\ln\frac{t_b' - t_b}{t_a' - t_a} = -UA \qquad \text{[IX-35]}$$

The rate of heat q_A, transferred on the total area A, may also be expressed by

$$q_A = UA\,(\Delta t)_m \qquad \text{[IX-36]}$$

where $(\Delta t)_m$ represents a mean temperature difference.

From Eqs. IX–35 and 36 it follows that

$$\frac{q_A}{(\Delta t)_m} = UA = -\frac{1}{N} \ln \frac{t_b' - t_b}{t_a' - t_a}$$

or

$$(\Delta t)_m = -\frac{q_A N}{\ln \dfrac{t_b' - t_b}{t_a' - t_a}}$$

Substituting for N from Eq. IX–32 gives

$$(\Delta t)_m = -\frac{\dfrac{q_A}{m_h c_h} + \dfrac{q_A}{m_c c_c}}{\ln \dfrac{t_b' - t_b}{t_a' - t_a}} \qquad \text{[IX–37]}$$

But

$$q_A = \int_{y=0}^{y=A} dq_h = \int_{y=0}^{y=A} dq_c$$

or by substitution from Eqs. IX–30 and 31

$$q_A = \int_{t'=t_a'}^{t'=t_b'} -m_h c_h\, dt' = \int_{t=t_a}^{t=t_b} +m_c c_c\, dt$$

and by integration

$$q_A = -m_h c_h (t_b' - t_a') = m_c c_c (t_b - t_a) \qquad \text{[IX–38]}$$

This substituted in Eq. IX–37 gives

$$(\Delta t)_m = -\frac{\dfrac{m_h c_h (t_a' - t_b')}{m_h c_h} - \dfrac{m_c c_c (t_a - t_b)}{m_c c_c}}{\ln \dfrac{t_b' - t_b}{t_a' - t_a}}$$

or

$$(\Delta t)_m = \frac{(t_a' - t_a) - (t_b' - t_b)}{\ln \dfrac{t_a' - t_a}{t_b' - t_b}} \qquad \text{[IX–39]}$$

This is known as the "log mean temperature difference." It is often written in the following form:

$$(\Delta t)_m = \frac{(\Delta t)_{\max} - (\Delta t)_{\min}}{\ln \dfrac{(\Delta t)_{\max}}{(\Delta t)_{\min}}} \qquad \text{[IX–40]}$$

In Eq. IX–40 $(\Delta t)_{max}$ and $(\Delta t)_{min}$ represent the maximum and minimum temperature differences in the heat exchanger. This simple relation for the log mean temperature difference may be used for the types of heat exchangers discussed. It must be altered for the more complex types of heat exchangers, such as the shell and tube devices with any number of passes on the shell side and tube side, and cross-flow exchangers with different pass arrangements and with mixed and unmixed flow. For these types correction factors have been worked out (Ref. IX–2), by which the log mean temperature difference for counter-current flow must be multiplied to give the true differences.

Example IX–5. A liquid-to-liquid counter-flow heat exchanger is used to heat a cold fluid from 120 F to 310 F. If the hot fluid enters at 500 F and leaves at 400 F, calculate the log mean temperature difference for the heat exchanger.

Solution:

$$(\Delta t)_{max} = 400 - 120 = 280 \text{ F} \quad \text{and} \quad (\Delta t)_{min} = 500 - 310 = 190 \text{ F}$$

Then according to Eq. IX–40

$$(\Delta t)_m = \frac{280 - 190}{\ln \dfrac{280}{190}} = 232 \text{ F}$$

IX–7 Some Applications

The analysis of the behavior of the various types of heat exchangers which include heat transfer by convection depends upon the knowledge of the film coefficients, the overall heat transfer coefficient, and the temperature differences. The values for U and h for the solution of a particular problem must either be taken from experimental results, industrial operation records, or from dimensional correlations. Some of the latter have been dealt with in Chapter VIII.

Example IX–6. Calculate the outside tube area for a single-pass steam condenser to handle 731,300 lb/hr dry saturated steam. The inlet steam pressure is 1.09 in. Hg and the hot-well temperature is 81.7 F. The cooling water enters the inside of the tubes at 61.4 F and leaves at 69.9 F. The tubes are of 1-in. outside diameter by 0.902-in. inside diameter, and the tube material has a thermal conductivity of 63 B hr^{-1} ft^{-1} F^{-1}. The average water velocity in each tube is 7 ft sec^{-1}. Assume that the steam side film coefficient is 1000 B hr^{-1} ft^{-2} F^{-1}.

Solution: The average water side coefficient has been determined in Ex. VIII–1, and was found to be $h = 1270$ B hr^{-1} ft^{-2} F^{-1}.

Since the values for the individual coefficients are known, the overall heat transfer coefficient may be calculated from Eq. IX–28a.

$$U_3 = \frac{1}{\dfrac{1}{1270(0.902)} + \dfrac{0.5 \ln 1/0.902}{12(63)} + \dfrac{1}{1000}} = 516 \text{ B hr}^{-1} \text{ ft}^{-2} \text{ F}^{-1}$$

In the further solution of the problem it will be assumed that the saturation temperature (81.7 F) corresponding to the pressure of 1.09 in. Hg prevails in the condenser. Then from Eq. IX–40

$$(\Delta t)_m = \frac{(81.7 - 61.4) - (81.7 - 69.9)}{\ln \dfrac{20.3}{11.8}} = 15.7 \text{ F}$$

The rate of heat energy required to condense dry saturated steam at 1.09 in. Hg and 81.7 F is equal to the product of the change in enthalpy and the weight rate of steam condensed. Thus

$$q = 731{,}300\ (1097.4 - 49.7) = 765{,}000{,}000 \text{ B/hr}$$

The area required according to Eq. IX–26 is

$$A_3 = \frac{765{,}000{,}000}{516(15.7)} = 94{,}500 \text{ ft}^2$$

Example IX–7. The following data were reported (Ref. IX–1) on a test of a shell and tube water-to-water heat exchanger with a total outside tube area of 48.1 ft². The arrangement is shown and the measured temperatures are given in Fig. IX–11. The heat exchanger consisted of 98 brass tubes having an outside diameter of ⅜ in. and a 0.049-in. wall. The shell, 6.06-in.

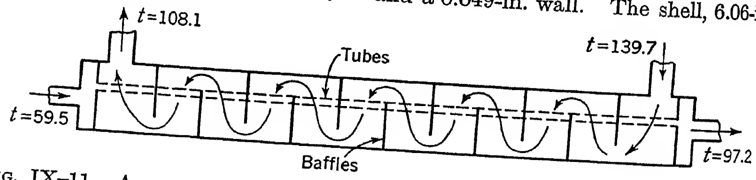


Fig. IX–11. Arrangement of a shell and tube heat exchanger with baffles (see Ex. IX–7).

inside diameter, had eleven half-moon baffles located approximately 4.3 in. apart. The dimensions and spacing of the baffles were such as to allow a minimum outside water passage area between the tubes of approximately 6.67 sq in. The rate of water flowing through the tubes and shell were 18,210 and 21,780 lb per hour respectively.

Calculate the overall heat transfer coefficient (a) from the test data, and (b) using the equations which have been established by correlation of data by dimensional analysis.

Solution: (*a*) In the derivation of the log mean temperature difference, differential elements were considered beginning with Eqs. IX-29 to 31. If instead of this the tube sections between any two adjacent baffles are assumed to be the elements, the derivation leads to the same result as can be easily proved. Therefore, in the present case, counter-flow is assumed to exist although this is not exactly true. With this assumption, according to Eq. IX-40,

$$(\Delta t)_m = \frac{(108.1 - 59.5) - (139.7 - 97.2)}{\ln \dfrac{(108.1 - 59.5)}{(139.7 - 97.2)}} = 46 \text{ F}$$

According to Eq. IX-26, the rate of heat transferred is

$$q = U_3(48.1)46$$

But according to Eq. IX-38

$$q = 18{,}210(1)\ (97.2 - 59.5) = 686{,}000 \text{ B/hr}$$

From the two last equations one obtains $U = 310$ B hr^{-1} ft^{-2} F^{-1}.

(*b*) The individual coefficient of heat transfer h_{34} for the outside of the tubes (heating fluid) will be calculated by means of Eq. VIII-8, considering now the flow in the shell as a flow normal to tube banks.

It will be assumed that the average tube temperature $t_3 = 113$ F. This will be checked again later.

According to Eq. VIII-7 the average film temperature is

$$= 123.9 \qquad\qquad = 118.4 \text{ F}$$

At this temperature the pertinent thermal properties of the water in consistent units are

$$\rho = \frac{61.8}{32.2} \text{ slug ft}^{-3} = 1.924 \text{ slug ft}^{-3}$$

$$\mu = 3.32(10^{-9}) \text{ lb ft}^{-2}\text{ hr} = 3.32(10^{-9})(3600)^2 \text{ slug ft}^{-1}\text{ hr}^{-1}$$
$$= 0.043 \text{ slug ft}^{-1}\text{ hr}^{-1}$$

$$c_p = 1 \text{ B lb}_m^{-1}\text{ F}^{-1} = 1(32.2) \text{ B slug}^{-1}\text{ F}^{-1}$$

$$k = 0.367 \text{ B hr}^{-1}\text{ ft}^{-1}\text{ F}^{-1}$$

Further one needs

$$D_3 = 0.375/12 = 0.03125 \text{ ft, and}$$

$$v_m = \frac{21{,}780/61.8}{6.67/144} = 7620 \text{ ft/hr}$$

Then

$$(Re) = \frac{v_m D_3 \rho}{\mu} = \frac{7620(0.03125)1.924}{0.043} = 10{,}630$$

$$(Pr) = \frac{\mu c_p}{k} = \frac{0.043(32.2)}{0.367} = 3.76$$

Substituting these dimensionless groups in Eq. VIII-8

$$(Nu) = \frac{h_{34} D_3}{k} = 0.33(10630)^{0.6}(3.76)^{1/3} = 133.7$$

and from this

$$h_{34} = \frac{133.7(0.367)}{0.03125} = 1570 \text{ B hr}^{-1}\text{ ft}^{-2}\text{ F}^{-1}$$

The individual coefficient h_{12} for the inside of any of the 98 tubes (heated fluid) will be determined from Eq. VIII-2.

At the average temperature of 78.3 F the thermal properties of water are

$$\rho = \frac{62.3}{32.2} \text{ slug ft}^{-3} = 1.94 \text{ slug ft}^{-3}$$

$$\begin{aligned}\mu &= 5.1(10^{-9}) \text{ lb ft}^{-2}\text{ hr} = 5.1(10^{-9})(3600)^2 \text{ slug ft}^{-1}\text{ hr}^{-1} \\ &= 0.066 \text{ slug ft}^{-1}\text{ hr}^{-1}\end{aligned}$$

$$c_p = 1 \text{ B lb}_m^{-1}\text{ F}^{-1} = 32.2 \text{ B slug}^{-1}\text{ F}^{-1}$$

$$k = 0.348 \text{ B hr}^{-1}\text{ ft}^{-1}\text{ F}^{-1}$$

Further

$$D_2 = 0.277/12 = 0.0231 \text{ ft}$$

$$v_m = \frac{18{,}210/62.3}{98\,\dfrac{\pi 0.0231^2}{4}} = 7120 \text{ ft/hr}$$

$$(Re) = \frac{v_m D_2 \rho}{\mu} = \frac{7120(0.0231)1.94}{0.066} = 4830$$

$$(Pr) = \frac{\mu c_p}{k} = \frac{0.066(32.16)}{0.348} = 6.09$$

By substitution in Eq. VIII-2

$$(Nu) = \frac{h_{12} D_2}{k} = 0.023(4830)^{0.8}(6.09)^{0.4} = 41.95$$

and from this

$$h_{12} = \frac{41.95(0.348)}{0.0231} = 632 \text{ B hr}^{-1}\text{ ft}^{-2}\text{ F}^{-1}$$

Substitution in Eq. IX-28 gives the overall heat transfer coefficient. Assume 58 B hr^{-1} ft^{-1} F^{-1} for the thermal conductivity of brass. Then from

Eq. IX-28*a*

$$\frac{0.03125}{0.0231}\frac{1}{632}+\frac{0.03125}{2}\frac{1}{58}\ln\frac{0.03125}{0.0231}+\frac{1}{1570}$$

$$=\frac{1}{0.00214+0.0000815+0.00064}=350\ \mathrm{B\ hr^{-1}\ ft^{-2}\ F^{-1}}$$

This value deviates from the actual value by 11.4 per cent of the latter. The deviation is not too great owing to the simplifying assumptions which had to be made. Considering that the three terms in the denominator of the last equation are proportional to the three heat resistances involved (see Eq. IX-20*a*) it is observed that the tube wall presents only about 3 per cent of the total resistance. Therefore, of the average temperature drop between the two fluids of about 45 F, only about 1.3 F is due to the heat resistance of the wall. Since t_3 is the average temperature of the outer surface of the tubes, that for the inner surface would be $t_3 - 1.3$. Then equating the heat flow between the heating water and the wall, and that between the wall and the heated water

$$- 123.9) = h_{12}A_2[78.3 - (t_3 - 1.3)]$$

or

$$1570(0.03125)\pi L(t_3 - 123.9) = 632(0.0231)\pi L[78.3 - (t_3 - 1.3)]$$

Evaluating gives $t_3 = 113.5$ F which is in excellent agreement with the value 113, assumed at the beginning. If there had been a considerable discrepancy, the calculation would have to be repeated with another estimate of t_3.

IX-8 Heat Transfer from a Wall at Uniform Temperature Suddenly Brought in Contact with a Medium at Different Temperature

In Sect. IV-3 a wall with an initial temperature of t_i whose surface temperature suddenly changed to t_s and remained constant thereafter was discussed. A wall at t_i the sides of which are suddenly exposed to a medium at t_0 will now be considered. In this case the surfaces do not immediately take on the temperature of the surroundings owing to the insulating effect of the fluid film next to the surfaces. As it was in the case of the thick slab, the complete solution of the problem is beyond the scope of this text. However, the final results are available as charts, for instance, those published by Groeber (Ref. IX-3). These are based on the following relations

$$t_s = t_0 + (t_i - t_0)\Phi_s\left(\frac{4\alpha\tau}{L^2}, \frac{hL}{2k}\right) \qquad \text{[IX-41]}$$

$$t_c = t_0 + (t_i - t_0)\Phi_c\left(\frac{4\alpha\tau}{L^2}, \frac{hL}{2k}\right) \qquad \text{[IX-42]}$$

$$Q = \rho c_p AL(t_i - t_0)\Psi\left(\frac{4\alpha\tau}{L^2}, \frac{hL}{2k}\right) \qquad \text{[IX-43]}$$

The symbols used are:

L = thickness of the wall.
A = area of one side of the wall.
ρ = density
c_p = specific heat
k = thermal conductivity
$\alpha = \dfrac{k}{\rho c}$ = thermal diffusivity } of the material of the wall.
t_i = initial uniform temperature of the wall.
t_0 = constant outer temperature, that is, the temperature of the surrounding medium with which the two sides of the wall suddenly come in contact.
τ = time elapsed after the wall comes in contact with the medium.
t_s = surface temperature of the wall at the time τ.
t_c = center temperature, that is, the temperature at the median plane of the wall at time τ.
h = heat transfer coefficient.
Q = heat energy released by the wall in the time interval τ.
Φ_s, Φ_c, Ψ = functions represented in Fig. IX–12 with the dimensionless groups $hL/2\,k$ and $4\alpha\tau/L^2$ as abscissa and parameter respectively.

It may be mentioned that with infinitely increasing τ the function Ψ approaches the value 1, so that according to Eq. IX–43 the heat released

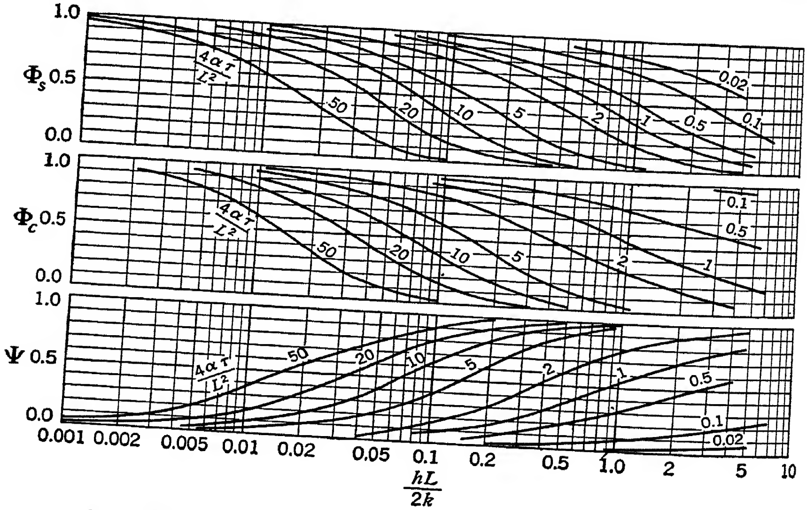


FIG. IX–12. The functions

in infinite time becomes

$$Q_\infty = \rho c_p AL(t_i - t_0)$$

This is easily seen because AL is the volume, ρAL the mass, and $\rho c_p AL$ is the heat capacity of the wall and $(t_i - t_0)$ the original temperature difference.

EXAMPLE IX-8. A wall 2 ft thick at an average initial temperature of 100 F is exposed on both sides to hot gases at 1000 F. If $k = 8$ B hr^{-1} ft^{-1} F^{-1}, $\rho = 162$ lb$_m$ ft^{-3}, $c_p = 0.3$ B lb$_m^{-1}$ F^{-1}, and $h = 1.6$ B hr^{-1} ft^{-2} F^{-1}, calculate (a) the temperature at the surface, (b) the temperature at the center plane of the wall, and (c) the heat absorbed by 10 sq ft of the wall 30 hours and 24 minutes after the gases come in contact with the wall.*

Solution:

$$\alpha = \frac{k}{\rho c_p} = \frac{8}{162(0.3)} = 0.1645 \text{ ft}^2 \text{ hr}^{-1}$$

$$\frac{4\alpha\tau}{L^2} = \frac{4(0.1645)30.4}{2^2} = 5$$

and

$$\frac{hL}{2k} = \frac{1.6(2)}{2(8)} = 0.2$$

From Fig. IX-12 the following values were obtained:

$$\Phi_s = 0.37, \quad \Phi_c = 0.41, \quad \Psi = 0.62$$

From Eq. IX-41

$$t_s = 1000 + (100 - 1000)0.37 = 667 \text{ F}$$

From Eq. IX-42

$$t_c = 1000 + (100 - 1000)0.41 = 631 \text{ F}$$

From Eq. IX-43

$$Q = (162)0.3\ (10)\ 2(100 - 1000)0.62 = -541{,}000 \text{ B}$$

The negative sign shows that the heat is not released but absorbed and stored by the wall.

* As mentioned in Sect. I-3, the density, related to pound mass, has the same numerical value as the specific weight related to pound force. Therefore $\gamma = 162$ lb ft^{-3} could have been given as well in the above example. But since the ordinary numerical values of c_p are related to pound mass, one must use ρ. The student should be encouraged to read, once more, the remarks in Sect. IV-1 concerning $C_p = \rho c_p$.

PROBLEMS

IX–1. For the conditions given in Ex. IX–1, calculate the rate of heat given up by the silver rod and the glass rod.

IX–2. For the conditions given in Ex. IX–1 except that the free end of the rod is uninsulated, show the temperature distribution in the axial direction by a graph for each of the three rods, and calculate the amount of heat given up. What fraction of the latter is lost through the free end?

IX–3. Calculate the overall heat transfer coefficient based on the outer area for a condenser tube ($k = 60$ B hr^{-1} ft^{-1} F^{-1}) having an outside and inside diameter of 1 in. and 0.902 in. respectively. Assume that the steam side and water side heat transfer coefficients are 1000 B hr^{-1} ft^{-2} F^{-1} and 800 B hr^{-1} ft^{-2} F^{-1} respectively.

IX–4. Calculate the overall heat transfer coefficient based on the inside area for a tube ($k = 58$ B hr^{-1} ft^{-1} F^{-1}) having an outside diameter of 2.375 in. and an inside diameter of 2.0 in. Assume that steam is condensing on the outside of the tube and that the individual coefficient for the steam side is 900 B hr^{-1} ft^{-2} F^{-1}. The coefficient for the inside is 1100 B hr^{-1} ft^{-2} F^{-1}.

IX–5. 200,000 lb/hr of water are heated in a parallel-flow heat exchanger from 230 F to 350 F. Hot gases used for heating the water enter the exchanger at 700 F and leave at 400 F. Determine the overall heat transfer coefficient if the total surface area is 20,000 ft^2.

IX–6. A parallel-flow heat exchanger is to be designed to heat 10,000 lb/hr of water from 70 F to 110 F with steam condensing at 250 F on the outside of the tubes. Tubes of 1-in. outside diameter and 0.9-in. inside diameter, 8 ft. long are available for use. If the average steam side coefficient is 1000 B hr^{-1} ft^{-2} F^{-1} and the thermal conductivity of the tube metal is 64 B hr^{-1} ft^{-1} F^{-1}, calculate the required number of tubes per pass and the number of passes. Assume that the velocity of the water entering each tube is equal to 1.5 ft/sec.

IX–7. Determine the heat absorbed by one square foot of a wall 1 ft thick initially at 80 F after the surfaces have been exposed to air at a temperature of 180 F for 3.5 hours. $k = 6$ B hr^{-1} ft^{-1} F^{-1}, $h = 12$ B hr^{-1} ft^{-2} F^{-1}, $c_p = 0.28$ B lb$_m^{-1}$ F^{-1}, and $\rho = 150$ lb$_m$ ft^{-3}.

IX–8. For the conditions given in Problem IX–7 calculate the temperatures at the center and surface of the wall.

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IX–2. R. A. Bowman, A. C. Mueller, and W. M. Nagle, "Mean Temperature Difference in Design," *Trans. Am. Soc. Mech. Eng.*, **63**, 283 (1940).

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* Title translated by the authors.

CHAPTER X

HEAT TRANSFER IN CONDENSING AND BOILING

X-1 Change of Physical Properties in Changes of Phase

When convection was dealt with in the former chapters, the assumption was made, although not explicitly stated, that the chemical composition and the phase of the fluid under consideration remained unchanged. In this chapter cases wherein a change of phase takes place will be briefly considered. The treatment will be restricted to condensation and boiling, processes of far-reaching importance in mechanical and chemical engineering. In these processes a sudden change from the gaseous to the liquid phase or the reverse change occurs, and by this the density, viscosity, specific heat, and thermal conductivity of the fluid change and heat energy becomes free or is absorbed. It is obvious that these changes will exert a tremendous influence upon the heat transfer which is connected with the process.

X-2 Condensation

Two types of condensation are known: one called dropwise, and the other film condensation. The former consists of the formation of drops on the cold surface which grow until removed from the surface under the influence of gravity and vapor friction. In film condensation a thin layer of condensate covers the cooling surface, increases in thickness, and flows down and off under the influence of the same forces as mentioned above.

In either type of condensation the heat of condensation which becomes free must be carried from the vapor to the cooling surface. Applying Eq. I-3, Δt is the temperature difference between the vapor and the solid surface, and h is a film coefficient of heat transfer which, however, only deserves this name for film condensation. Coefficients for dropwise condensation are considerably higher than are those for film condensation. For steam the film coefficient for dropwise condensation has been observed to be fifteen to twenty times larger than for film condensation (Ref. X-1). This, however, relates only to the individual heat transfer on the steam side. The overall coefficient will increase much less because according to Eq. IX-21 it is controlled by the smallest individual coefficient, which for this case would be on the

side of the cooling liquid. Dropwise condensation is unstable and, until this disadvantage has been overcome, it is unsuited in practice for condenser design work. It is a matter of interest, however, to know something about these two types of condensation (Ref. X–2).

Film condensation occurs when clean steam condenses on a clean polished or rough surface.

If the surface is contaminated by materials such as mercaptans or fatty acids, dropwise condensation of steam may occur.

Often both types of condensation occur at the same time on the same surface; this process is known as mixed condensation.

W. Nusselt developed in 1916 (Ref. X–3) a simple theory of film condensation, the main features of which are given as follows:

A vapor condensing in a film on a cooling surface flows on the surface under the influence of gravity and vapor friction, but is retarded by the viscous forces of the liquid. The heat of condensation passes through the film from the condensing vapor to the wall. Because the thermal conductivity of liquids is small, the film, thin as it may be, presents a rather great resistance to the flow of heat, and, therefore, an appreciable temperature drop exists across the film. The temperature of the surface of the film in contact with the wall is equal to that of the wall, and the surface of the film in contact with the vapor is assumed to be at the saturation temperature. By combining the laws of laminar flow of a fluid and of heat conduction through it, Nusselt arrived at the following relation:

$$h = C\left(\frac{g\rho^2 l k^3}{L\mu \cdot \Delta t}\right)^{1/4} \qquad \text{[X-1]}$$

wherein, using foot, hour, and slug units

h = the average coefficient of heat transfer, taken for the whole condensing surface, in B hr^{-1} ft^{-2} F^{-1};
g = the gravitational constant $418(10^6)$ft hr^{-2};
l = the latent heat of evaporation in B/slug, taken at saturation temperature;
L = the height of a vertical wall or the outer diameter of a horizontal tube on which the condensation occurs;
ρ, μ, and k = the density, dynamic viscosity, and thermal conductivity of the condensate at its average temperature;
C = 0.943 for vertical walls; C = 0.725 for horizontal tubes.

This equation is valid for arbitrary vapors which are practically at rest, that is, no artificial means are used to move the vapor. For forced convection of condensing vapors see Ref. X–3.

EXAMPLE X-1. Determine the heat transfer coefficient for steam at 142 F condensing on the outside of a horizontal 1-in. diameter cylinder at 138 F.

Solution: According to the steam tables (Ref. VIII-2) and Tables II-2 and VI-1, the physical properties taken at the average temperature of 140 F are

$$\gamma = 61.3 \text{ lb/ft}^3 \text{ corresponding to } \rho = \frac{61.3}{32.2} = 1.90 \text{ slug/ft}^3,$$

$$l = 1014 \text{ B/lb or } l = 1014(32.2) = 32{,}600 \text{ B/slug},$$

$$k = 0.377 \text{ B hr}^{-1}\text{ ft}^{-1}\text{ F}^{-1},$$

$$\mu = 2.73(10^{-9}) \text{ lb ft}^{-2}\text{ hr or } \mu = 2.73(10^{-9})\ (3600)^2$$

$$= 0.0354 \text{ slug ft}^{-1}\text{ hr}^{-1}.$$

By substitution in Eq. X-1

$$h = 0.725\left[\frac{(418)10^6(1.90)^2\ 32{,}600(0.377)^3}{(1/12)\ 0.0354(142 - 138)}\right]^{1/4}$$

$$= 0.725(3865) = 2800 \text{ B hr}^{-1}\text{ ft}^{-2}\text{ F}^{-1}$$

X-3 Boiling

A glance into a kettle of boiling water shows that boiling is a process of heat transfer by pronounced convection. Until recently, very little was known as to the actual nature of this process, notwithstanding the fact that engineers have been designing and constructing boilers for more than a hundred years. There are two kinds of vaporization analogous to the two types of condensation, the one is called film boiling, the other nuclear boiling. The former corresponds to film condensation, but usually does not occur in practice. It is observed when heating is so intense that a vapor layer is formed on the heating surface. Such a layer, owing to the low heat conductivity of gases, forms a resistance to the heat flow, and by this the temperature of the heating surface increases rapidly. At times, this type of boiling becomes dangerous. The phenomenon of water drops dancing on a very hot plate without evaporating is due to this film effect.

The usual kind of vaporization is nuclear boiling, that is, boiling which starts from nuclei, tiny cells of vaporization, such as air bubbles which were dissolved in the liquid or absorbed on the heating surface before the heating took place. Such gas cells generally originate on the small natural roughnesses of heating surfaces, and for this reason boiling always begins on these surfaces.

Investigations of M. Jakob, performed in the last decade with different co-workers (Ref. X-3 and 4), have revealed the mechanism of nuclear boiling as follows.

From the solid heating surface the heat energy is transmitted b convection to the liquid in contact with that surface and then from th liquid to the surface of the tiny bubbles which form on the solid su face. At the bubble surfaces, more liquid is vaporized by the incomin heat flow, and the bubble grows until by its own buoyancy it break off from the surface and rises in the liquid. In this movement, mor evaporation takes place on the bubble surface; it increases in risin and finally blows up at the level which separates the liquid from th vapor space above it. This mechanism is only possible if temperatur differences exist by which the heat is driven in the described way Indeed Jakob's experiments have proved that the temperature of a boiling liquid is always slightly higher than the saturation temperature which exists inside the vapor bubbles and above the liquid level. Furthermore, the heating surface is warmer than the liquid.

As a consequence of the fact that heat energy in a boiling liquid generally makes a detour from the heating surface over the liquid to the vapor bubbles, the experimental results were represented by formulas which are identical with those dealt with in Chapters VI and VII for free convection between a solid surface and a fluid. This is valid up to a certain limit which is at $(Gr)(Pr) \approx 10^8$ to 10^9. Below this limit the movement of the vapor bubbles is of secondary influence as compared to the convection between the heating surface and the liquid. Above this limit the columns of rising vapor bubbles act like liquid stirrers and forced convection takes place with much more intensive heat transfer and vaporization. Research in this range is still in progress. Some of the more recent results will be found in Ref. X–3, 5, 6, and 7.

Denoting the rate of heat flow per unit heating surface by q'' and the unit value of this magnitude by $q_0'' = 1$, both in B hr^{-1} ft^{-2}, the following formulas are recommended for water boiling at standard atmospheric pressure p_s under free convection conditions. In all of these equations the units for h are B hr^{-1} ft^{-2} F^{-1}.

For horizontal heating surfaces in wide vessels up to $q'' = 5000$ B hr^{-1} ft^{-2}:

$$h_h = 43 \left(\frac{q''}{q_0''}\right)^{1/4} \qquad \text{[X-2]}$$

from $q'' = 5000$ to 75,000 B hr^{-1} ft^{-2}:

$$h_h = 0.64 \left(\frac{q''}{q_0''}\right)^{3/4} \qquad \text{[X-3]}$$

For the inside of vertical tubes:

$$h_t \approx 1.25\, h_h \qquad \text{[X-4]}$$

For vertical heating surfaces in wide vessels up to $q'' = 1000$ B hr^{-1} ft^{-2}:

$$h_v = 50 \left(\frac{q''}{q_0''}\right)^{1/8} \qquad \text{[X-5]}$$

from $q'' = 1000$ to 20,000 B hr^{-1} ft^{-2}:

$$h_v = 0.7 \left(\frac{q''}{q_0''}\right)^{3/4} \qquad \text{[X-6]}$$

The influence of pressure p can be considered approximately by using the formula

$$h = h_s \left(\frac{p}{p_s}\right)^{1/4} \qquad \text{[X-7]}$$

wherein the subscript s relates to standard atmospheric conditions.*

It may seem strange that in the above equations q'' is divided by a magnitude q_0'' which is equal to 1. This was done to retain the dimensional soundness of the equations. q''/q_0'' is independent of the system of units used. Therefore, the constant factor of the equations, for instance, 50 in Eq. X–5, is the value of the coefficient h for $q'' = q_0'' = 1$ B hr^{-1} ft^{-2}. This formal statement, however, does not mean that the equation is valid down to $q'' = 1$.

If it is desired to convert the equation into metric units, it is only necessary to convert the constant factor into kcal hr^{-1} m^{-2} C^{-1} and to convert q'' and q_0'' to kcal hr^{-1} m^{-2} units, so that Eq. X–5 becomes

$$h_v = \frac{50}{0.205} \left(\frac{q''}{q_0''}\right)^{1/8} = 244 \left(\frac{q''}{q_0''}\right)^{1/8} \qquad \text{[X-8]}$$

in the metric system. Using $h_v = 50(q'')^{1/8}$ instead of Eq. X–5 and then simply converting h_v and q'' into metric units would lead to erroneous results as can easily be checked.

* Eqs. X–2, 5, and 6 have been set up by M. Jakob based on unpublished correlations of his own and some other experiments (see Ref. X–3 and 4), particularly those of Insinger and Bliss (Ref. X–6). Equation X–3 has been given by Fritz (Ref. X–8) and is based on experiments of Jakob and Fritz, and Jakob and Linke. Equation X–4 is a rough estimate by Jakob (Ref. X–5). According to the latter paper, the influence of pressure from $p = 1.8$ to $p = 14.7$ lb/sq in. could be considered approximately by multiplying h for atmospheric pressure p_s with $\left(\frac{p}{p_s}\right)^{1/3}$ and between $p = 14.7$ and $p = 226$ lb/sq in. by multiplying with $\left(\frac{p}{p_s}\right)^{1/6}$. Equation X–7 is due to Bonilla and Perry (Ref. X–7). The exponent $\frac{1}{4}$ in their equation is just the mean of the exponents $\frac{1}{3}$ and $\frac{1}{6}$.

Because

$$q'' = h(\Delta t) \qquad \text{[X-9]}$$

by definition where Δt is the temperature difference between the heating surface and the boiling liquid, it is easy to express h as a function of Δt, instead of q'', as has been done in Eqs. X-2 to 8.

PROBLEMS

X-1. Compute the heat transfer coefficient for steam at 160 F condensing on the outside of a horizontal 3-in. diameter cylinder at 154 F.

X-2. Determine the heat transfer coefficient for steam at 142 F condensing on a vertical plate 6 in. high at 138 F.

X-3. Calculate the approximate time required to condense 500 lb of steam at 150 F on a horizontal cylinder 1 in. in diameter and 3 ft long at 144 F.

X-4. Determine the value of the heat transfer coefficient h in kcal hr^{-1} m^{-2} C^{-1} units for steam condensing at 95 C on a horizontal pipe 4 cm in diameter. Assume that the temperature difference is 10 C.

X-5. If the rate of heat input to the horizontal surface of an apparatus used for evaporating water under standard atmospheric conditions, is 2800 B hr^{-1} ft^{-2}, calculate the heat transfer coefficient.

X-6. Solve Problem X-5 if the heat input had been 60,000 B hr^{-1} ft^{-2}.

X-7. The heat input based on the outer surface area of a vertical thin wall pipe 2 in. in diameter is 3000 B/hr for 5 ft of pipe. If water flowing slowly through the tube is being evaporated under standard atmospheric conditions, estimate the heat transfer coefficient for the boiling liquid.

X-8. Calculate the heat transfer coefficient between a vertical plate and water boiling at standard atmospheric pressure if the heat input is 600 B hr^{-1} ft^{-2}.

X-9. The heat input to a vertical wall of a device for boiling water is 15,000 B hr^{-1} ft^{-2}. If the pressure in the apparatus is five atmospheres absolute, calculate the approximate heat transfer coefficient between the wall and the boiling liquid.

X-10. If the heat transfer coefficient between a vertical plate and boiling water is 500 B hr^{-1} ft^{-2} F^{-1}, estimate the rate of heat input to the plate, assuming standard atmospheric conditions.

X-11. Calculate the heat transfer coefficient in metric units between a vertical tube and boiling water if the rate of heat input is 8000 kcal hr^{-1} m^{-2}, assuming standard atmospheric conditions.

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* Title translated by the authors.

CHAPTER XI

HEAT TRANSFER BY RADIATION

XI–1 Heat Radiation — A Type of Wave Motion

Whenever heat flows from one part of a body to another by conduction the intervening material of the body becomes heated, that is, the temperature of the body will vary from the hot side to the cold side according to a definite relation, depending upon the position in the body. Radiant heat transfer differs from heat flow by conduction in that the medium through which the flow takes place does not become heated. The earth receives radiant energy from the sun, the radiation passing through the intervening cold empty space. The warmth one feels from an open camp fire on a chilly autumn evening is due chiefly to the radiation from the fire to the body. This energy is transferred through the cool night air without heating it up appreciably.

The transfer mechanism of radiant heat energy is complicated, but a general understanding may be had by considering the heat transfer from the sun to the earth. The thermal energy of the sun is first converted into an electro-magnetic wave motion called radiation which travels from the sun to the earth in about eight minutes and is there absorbed, raising the temperature of the absorbing medium. Thus, the mechanism of energy transfer by radiation is composed of three distinct components: first, the conversion of the thermal energy of the hot source into an electro-magnetic wave motion; second, the passage of the wave motion through the intervening space; and, third, the reconversion of the wave motion into thermal energy by absorption at the cold body.

Radiant energy is the same type of wave motion as radio waves, x rays, and light waves except for the wavelength. It is apparent that radiant heat energy is governed by exactly the same laws as light: it travels in straight lines, obeys the laws of reflection, suffers refraction, may be polarized, and is weakened with the inverse square of the radial distance from the source of radiation. The velocities of light and radiant energy waves are the same, that is, about 186,000 miles per second. For purposes of comparison some well-known electro-magnetic waves are given in Table XI–1.

TABLE XI-1

ELECTRO-MAGNETIC WAVES

Name	Wavelength Range in Microns*
Cosmic rays	up to $1(10^{-6})$
Gamma rays	$1(10^{-6})$ to $140(10^{-6})$
X rays	$6(10^{-6})$ to $100{,}000(10^{-6})$
Ultraviolet rays	0.014 to 0.4
Visible or light rays	0.4 to 0.8
Infrared or heat rays	0.8 to 400
Radio	$10(10^{6})$ to $30{,}000(10^{6})$

*1 micron = 10^{-6} meter.

XI-2 The Concept of a Perfect Black Body

When radiant energy falls on a body, part may be absorbed, part reflected, and the remainder transmitted through the body. In mathematical form,

where α = absorptivity or the fraction of the total energy absorbed,
ρ = reflectivity or the fraction of the total energy reflected,
τ = transmissivity or the fraction of the total energy transmitted through the body.

For the majority of opaque solid materials encountered in engineering, except for extremely thin layers, practically none of the radiant energy is transmitted through the body. If the discussion is limited to opaque bodies Eq. XI-1 becomes

$$\alpha + \rho = 1 \qquad \text{[XI-2]}$$

An arrangement which will absorb all the radiant energy at all wavelengths and reflect none is called a perfect black body. Actually no material with $\alpha = 1$ and $\rho = 0$ exists. Even the blackest surfaces occurring in nature still have a reflectivity of about 1 per cent ($\rho = 0.01$). The physicist G. Kirchhoff, however, conceived the following possibility of making a practically perfect black body. If a hollow body is provided with only one very small opening, and is held at uniform temperature, then any beam of radiation entering by the hole is partly absorbed, and partly reflected inside. The reflected radiation will not find the outlet, but will fall again on the inside wall. There it will be only partly reflected and so on. By such a sequence of reflections the entering radiation will be weakened so much that almost no part of it will leave the hole. Thus the area of the hole is like a perfectly ab-

sorbing surface, and an arrangement of this kind will act just as a perfect black body. It may be considered a measure by which the absorptivity of any substances may be determined.

XI–3 Planck's Law of Monochromatic Radiation of a Black Body

All substances emit radiation, the quality and quantity depending upon the absolute temperature* and the properties of the material composing the radiating body.

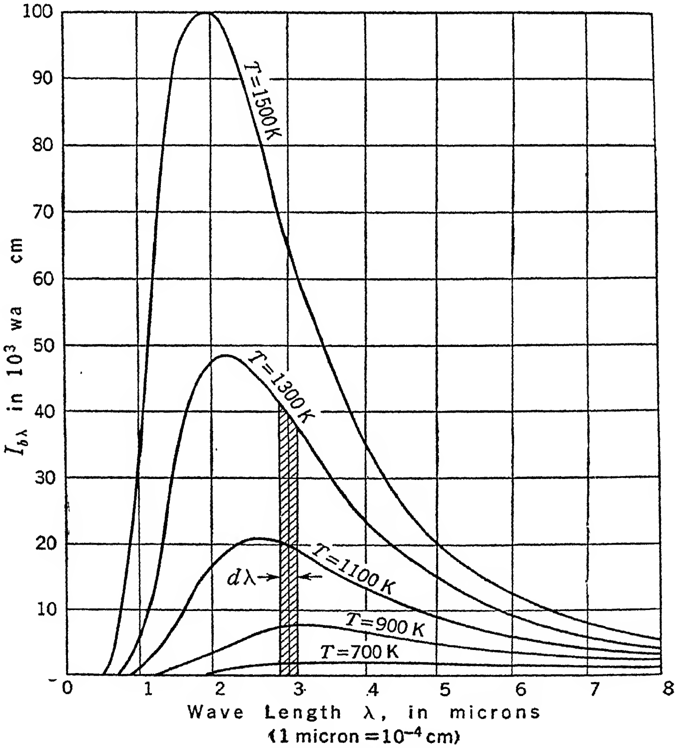

FIG. XI–1. Monochromatic intensity of radiation for a black body at various absolute temperatures (Planck's law).

* As far as radiation depends on the temperature of the radiating body, it is called temperature radiation. There are other kinds of radiation which, however, are not of the general importance of temperature radiation and, therefore, will not be dealt with in this book.

The total radiant energy emitted by a surface per unit area per unit time will be called the total density of emission and denoted by the symbol E. It can be considered as an integral of differential values, dE_λ, according to the equation

$$\int_{\lambda=0}^{\lambda=}$$

taken over the whole range of wavelengths from $\lambda = 0$ to $\lambda = \infty$. By this equation another magnitude, I_λ, is defined which is called the monochromatic intensity of radiation, the subscript relating to the wavelength λ of the radiation. For a black body, the respective symbols are E_b and $I_{b\lambda}$. The latter magnitude depends on the temperature of the radiating surface and the wavelength of radiation according to Planck's law of monochromatic radiation. This is represented graphically in Fig. XI–1, with the wavelengths as abscissas and the absolute temperature as parameter of the single curves. E_b and $I_{b\lambda}$ are connected by the relation

The areas between the curves and the axis of abscissas from $\lambda = 0$ to $\lambda = \infty$ give the respective total flux densities E_b in $[10^{-1}$ watt $\mathrm{cm}^{-2}]$.

It may be mentioned that a few gases such as carbon dioxide and steam in thick layers radiate like a black body, but not at all wavelengths, as shown in Fig. XI–1, but only in some narrow wavelength intervals, called bands of gas radiation. The treatment of gas radiation is beyond the scope of this text.

XI–4 Kirchhoff's Law of Radiation

In order to develop a relation between the emissive and absorptive power of a surface, first consider a large heated enclosure wherein the radiation is between $\lambda + d\lambda$ and λ. After the equilibrium temperature has been established, it is permissible to assume that the black body condition exists within the enclosure. A second body, having the absorptivity α_λ and emitting radiation with the intensity I_λ, is placed within the cavity of the large body. When the equilibrium condition has been reestablished, the amount of energy absorbed by the small body will be equal to the energy emitted in the same time. Let dq represent the rate of energy at wavelength λ striking the enclosed body. The rate of absorbed energy will be equal to $\alpha_\lambda \cdot dq$. By definition the rate of emitted energy will be equal to $A_1 I_\lambda \cdot d\lambda$ where A_1 is the area of the enclosed body. Since the two rates of energy are equal

$$\alpha_\lambda \cdot dq = A_1 I_\lambda \, d\lambda \qquad \text{[XI–4]}$$

For a black body this equation would become

$$1 \cdot dq = A_1 I_{b\lambda}\, d\lambda \qquad [XI\text{-}4a]$$

From Eqs. XI–4 and 4a

$$I_\lambda = \alpha_\lambda I_{b\lambda} \qquad [XI\text{-}5]$$

Introducing a new term, the emissivity ϵ_λ of a surface, by the definition

$$\frac{I_\lambda}{I_{b\lambda}} = \epsilon_\lambda \qquad [XI\text{-}6]$$

it follows that

$$\epsilon_\lambda = \alpha_\lambda \qquad [XI\text{-}7]$$

Any one of the last three relations represents Kirchhoff's law. Expressed in words, Eq. XI–5 means that the rate of radiant energy, which is emitted by a surface at any temperature and in a small range of wavelengths, is found from the known rate of energy which, under the same conditions, would be emitted from a black surface, by multiplying with the absorptivity.

Because the absorptivity, according to Eq. XI–2, must lie between 0 and 1, it is apparent that a black body is the best emitter of energy. Equation XI–7 further shows that a good emitter is also a good absorber and that emissivity and absorptivity are identical properties of a surface.

By integrating Eqs. XI–4 and 4a over the whole range of wavelengths $\lambda = 0$ to $\lambda = \infty$, and considering Eqs. XI–3 and 3a, it is easily seen that Kirchhoff's law holds for the total radiation as well as for monochromatic radiation. In terms of total radiation it is usually expressed in one of the following forms:

$$\epsilon = \alpha \qquad [XI\text{-}7a]$$

Since perfectly black substances are not attainable in engineering practice, the equation established for black body radiation must be modified if other bodies are to be considered. For convenience the emissive power of an ordinary opaque body will be referred to that of a black body by the emissivity ϵ.

XI–5 Stefan-Boltzmann's Law of Total Radiation

In 1879 J. Stefan concluded from experimental data that the total energy emitted by a black body is proportional to the fourth power of the absolute temperature of the body. Five years later L. Boltzmann derived the same law from a theoretical thermodynamic standpoint. It likewise can be derived from Planck's law by integration or by

metering the areas below the curves in Fig. XI-1 from $\lambda = 0$ up to $\lambda = \infty$. In honor of its discoverers the law has been called the Stefan-Boltzmann law. It may be written either in the form

$$E_b = \sigma T^4 \tag{XI-8}$$

or by

$$q_b = \tag{XI-9}$$

where A = the radiating area
T = the absolute temperature and
σ = Stefan-Boltzmann's natural constant.

In British technical units, according to the best known measurements,

$$\sigma = 0.174(10^{-8}) \text{ B hr}^{-1} \text{ ft}^{-2} \text{ R}^{-4}$$

For convenience Eq. XI-9 may be written as

[XI-10]

Using this presents the advantage that the term raised to the fourth power gives relatively low figures in numerical calculations. Equations XI-8, 9, and 10 hold for black surfaces only.

If the emissivity of a body can be considered as independent of the wavelength and temperature, which fortunately is approximately true for most non-metallic surfaces in the ordinary temperature range, the body is called a gray radiator. For this kind of radiator, instead of Eq. XI-9, the following generalization of Stefan-Boltzmann's law can be used

$$q_g = \epsilon_g \sigma A T^4 \tag{XI-11}$$

where subscript g means gray radiator and ϵ_g is the emissivity of the gray surface. Still more general is the form

$$q = \epsilon \sigma A T^4 \tag{XI-12}$$

This formula has been mentioned in Chapter I as Eq. I-4. It can be used for any surfaces; ϵ, however, will not be a constant but a function of T in the general case.

XI-6 The Emissivity or Absorptivity of Different Bodies

The emissivity or absorptivity of a body has an importance in radiation similar to that of heat conductivity in conduction. They are factors of proportionality in the basic Eqs. I-4 and 1 respectively.

Generally, it is difficult if not impossible to estimate or indicate the emissivity of a surface with an accuracy of a few per cent because the

emissivity depends to some extent on the behavior of the surface, particularly as far as metallic surfaces are concerned.

The emissivity of non-metallic bodies does not vary much at ordinary temperatures. Some approximate values are given in Table XI-2.

TABLE XI-2

EMISSIVITY ϵ OF NON-METALLIC BODIES AT ORDINARY TEMPERATURES

Materials	ϵ
Iron oxide, carbon, oil	0.80
Rubber (gray, soft), wood (planed), paper	0.85 to 0.90
Roofing paper, enamel, lacquer, porcelain (glazed)	
Fused quartz (rough), brick (red, rough)	0.91 to 0.94
Marble (gray, polished), glass (smooth)	
Asbestos slate (rough), lampblack-waterglass, ice, water	0.95 to 0.99

It is seen that smooth and rough surfaces (like polished marble and rough brick) have almost the same emissivity, and that ice and water are close to black body conditions, as is lampblack. This, however, holds only for low-temperature radiation. According to Fig. XI-1 the maximum of intensity of radiation shifts to greater wavelengths with decreasing temperature. Thus radiation at low temperature is mainly long-wave radiation. If short-wave radiation, like that of the sun, strikes ice or a white surface the absorptivity is much smaller than when

TABLE XI-3

EMISSIVITY ϵ OF POLISHED METALLIC SURFACES

Metal	Temperature		
	100 F	500 F	1000 F
Aluminum	0.04	0.05	0.08
Copper	0.04	0.05	0.08
Gold	0.02	0.02	0.03
Silver	0.01	0.02	0.03
Steel	0.07	0.10	0.14

it hits a black surface. The good reflection of sunlight by ice or white fabrics is well known. It is less well known that the reflection of the same bodies is very small for long-wave radiation. So, if a quantity of soil is mixed with lampblack and another is mixed with chalk, it may be found that the black soil will be 10 to 12 F warmer than the white soil when exposed to the radiant energy of the sun. After sunset, how-

ever, white soil will cool as fast as black soil if the issuing temperature is the same, indicating that there is no appreciable difference in the radiating properties of black or white soil for long-wave radiation.

Metallic smooth surfaces emit very little radiation at ordinary temperature, and the emissivity increases moderately at higher temperature. A few values are given in Table XI-3.

However, when the surfaces are oxidized, i.e., covered with a chemically non-metallic layer, the emissivity increases enormously.

For instance, heavily oxidized aluminum may have $\epsilon \approx 0.85$ at 100 F and $\epsilon \approx 0.50$ at 1000 F; black oxidized copper or cast iron with cast skin or oxidized steel may have $\epsilon \approx 0.80$ at ordinary temperature.

XI-7 Heat Exchange by Radiation Between Large Parallel Black Planes

Thus far, emission or absorption of a surface has been discussed without considering its exchange of radiation energy with other surfaces. In practical cases, however, there is always such an exchange, and this often complicates the situation very much.

The reason one feels warm when standing in front of an open fireplace is because the fireplace radiates more energy to the body than the body radiates to the fireplace. Engineering problems of radiation must be dealt with in a similar way.

Consider, for instance, two large parallel planes. It may be assumed that the surfaces are perfectly black and so large that the influence of the edges will be negligible, that is, as though the areas were infinitely large. Then obviously all the energy radiated by one plane will be received by the other. If the absolute temperature of the warmer plane is T_1, then the rate of energy emitted per unit area is

$$= \sigma T_1^4$$

Likewise, the rate of energy emitted by the colder plane at absolute temperature T_2 is

$$E_{b2} = \sigma T_2^4$$

Since all the energy received by either plane is completely absorbed, the net interchange between the two surfaces is

$$q_{12}'' = E_{b1} - E_{b2} = \sigma(T_1^4 - T_2^4) \qquad \text{[XI-13]}$$

As formerly, q means the rate of heat flow, and the double prime sign indicates that unit areas are considered, whereas the double subscript (12) relates to the net heat exchange between the surfaces 1 and 2 and the direction of the heat flow from 1 to 2.

EXAMPLE XI–1. Calculate the net radiant interchange between unit areas of two parallel perfectly black planes, infinite in extent, at temperatures of 800 F and 1200 F respectively.

Solution:

$$T_1 = 460 + 1200 = 1660 \text{ R} \quad T_2 = 460 + 800 = 1260 \text{ R}$$

Substituting in Eq. XI–13 gives

$$q_{12}'' = 0.174 \left[\left(\frac{1660}{100}\right)^4 - \left(\frac{1260}{100}\right)^4\right]$$

$$= 8{,}850 \text{ B hr}^{-1} \text{ ft}^{-2}$$

XI–8 Heat Exchange by Radiation Between Large Parallel Planes of Different Emissivity

The rate of energy emitted by unit area of plane 1 at absolute temperature T_1 and emissivity ϵ_1 is $E_1 = \epsilon_1 \sigma T_1^4$. Of this energy the part $\epsilon_2 E_1 = \epsilon_2 \epsilon_1 \sigma T_1^4$ will be absorbed by plane 2. The difference $E_1 - \epsilon_2 E_1$ represents the amount reflected by 2 since the transmissivity τ is assumed to be zero. Part of the energy reflected from plane 2 will be absorbed at plane 1 and the remainder reflected back to plane 2 and so on. By similar reasoning the density of radiation $E_2 = \epsilon_2 \sigma T_2^4$ may be traced through the various absorptions and reflections (Ref. XI-1).

The rates of all amounts of energy emitted or reflected minus those absorbed per unit area by plane 1 may be represented by the series:

$$\begin{aligned} q_{12}'' = \; & \epsilon_1 \sigma T_1^4 [1 - \epsilon_1 (1 - \epsilon_2) - \epsilon_1 (1 - \epsilon_2)(1 - \epsilon_1)(1 - \epsilon_2) \\ & - \epsilon_1 (1 - \epsilon_2)(1 - \epsilon_1)^2 (1 - \epsilon_2)^2 - \cdots] \\ & - \epsilon_2 \sigma T_2^4 [\epsilon_1 + \epsilon_1 (1 - \epsilon_1)(1 - \epsilon_2) + \epsilon_1 (1 - \epsilon_1)^2 (1 - \epsilon_2)^2 + \cdots] \end{aligned}$$

or introducing the symbol

$$z = (1 - \epsilon_1)(1 - \epsilon_2) \qquad \text{[XI-14]}$$

$$\begin{aligned} q_{12}'' = \; & \epsilon_1 \sigma T_1^4 [1 - \epsilon_1 (1 - \epsilon_2) - \epsilon_1 (1 - \epsilon_2) z - \epsilon_1 (1 - \epsilon_2) z^2 - \cdots] \\ & - \epsilon_2 \sigma T_2^4 [\epsilon_1 + \epsilon_1 z + \epsilon_1 z^2 + \cdots] \end{aligned} \qquad \text{[XI-15]}$$

But because $z < 1$,

$$1 + z + z^2 + z^3 + \cdots = \frac{1}{1 - z}$$

Substituting this in Eq. XI–15

$$q_{12}'' = \epsilon_1 \sigma T_1^4 \left[1 - \frac{\epsilon_1 (1 - \epsilon_2)}{1 - z}\right] - \epsilon_2 \sigma T_2^4 \frac{\epsilon_1}{1 - z}$$

Resubstituting z from Eq. XI-14 and simplifying

$$q_{12}'' = \frac{\sigma}{\dfrac{1}{\epsilon_1} + \dfrac{1}{\epsilon_2} - 1}(T_1^4 - T_2^4) \qquad \text{[XI-15}a\text{]}$$

This is the net heat exchange between the planes because it is the excess of emission and reflection of plane 1 above its absorption.

EXAMPLE XI-2. Calculate the net radiant interchange per square foot for two very large planes at temperatures of 1000 F and 600 F respectively. Assume that the emissivity of the hot plane is 0.9 and for the cold plane 0.7.

Solution:

$$T_1 = 460 + 1000 = 1460 \text{ R}$$
$$T_2 = 460 + 600 = 1060 \text{ R}$$

According to Eq. XI-15*a*

$$q_{12}'' = \frac{0.174}{\dfrac{1}{0.9} + \dfrac{1}{0.7} - 1}(14.6^4 - 10.6^4) = 3710 \text{ B hr}^{-1}\text{ ft}^{-2}$$

XI-9. Heat Exchange by Radiation Between an Enclosed Body and the Enclosure.

The radiant interchange between a body 1 having a complete convex surface and a concave surface 2 which surrounds it may be determined by the following relation, derived first by C. Christiansen in 1883:

$$q_{12} = \sigma A_1(T_1^4 - T_2^4)\frac{1}{\dfrac{1}{\epsilon_1} + \dfrac{A_1}{A_2}\left(\dfrac{1}{\epsilon_2} - 1\right)} \qquad \text{[XI-16]}$$

In this relation the subscript 1 refers to the enclosed body and 2 to the enclosing surface. A_1 and A_2 are the respective surface areas.

EXAMPLE XI-3. Pentane at −221 F is to be stored in the inner of two concentric polished brass spheres 9 in. and 12 in. in diameter respectively. If the emissivity of the polished brass is 0.03, calculate the total radiant heat interchange between the spheres. Neglect the temperature drop through the metal and assume that the outer sphere metal temperature is 70 F.

Solution:

$$A_1 = 4\pi r_1^2 = 4\pi\left(\frac{4.5}{12}\right)^2 = 1.77 \text{ ft}^2$$

$$A_2 = 4\pi r_2^2 = 4\pi\left(\frac{6}{12}\right)^2 = 3.14 \text{ ft}^2$$

$$T_1 = 460 - 221 = 239 \text{ R}$$

$$T_2 = 460 + 70 = 530 \text{ R}$$

According to Eq. XI–16

$$q_{12} = (0.174)1.77(2.39^4 - 5.30^4)\frac{1}{\dfrac{1}{0.03} + \dfrac{1.77}{3.14}\left(\dfrac{1}{0.03} - 1\right)}$$

$$= -4.53 \text{ B/hr}$$

The negative sign of the result indicates that heat is flowing from surface 2 to 1.

XI–10 Heat Exchange by Radiation Between a Small Enclosed Body and the Enclosure

Assuming that the area A_2 of the enclosure is large compared with the area A_1 of the enclosed body, Eq. XI–16 simplifies to

$$q_{12} = \epsilon_1 \sigma A_1 (T_1^4 - T_2^4) \qquad \text{[XI-17]}$$

As in Eq. XI–16 the enclosed body must be convex throughout, and the enclosure concave throughout.

Example XI–4. A 2-in. oxidized iron pipe at 300 F passes through a room in which the surroundings are at a temperature of 80 F. If the emissivity of the pipe metal is 0.8, calculate the net interchange of radiant energy per foot length of pipe.

Solution:

$$T_1 = 460 + 300 = 760 \text{ R}; \quad T_2 = 460 + 80 = 540 \text{ R}$$

The surface area of one foot of 2-in. standard pipe is

$$A_1 = \pi 2.375/12 = 0.624 \text{ ft}^2$$

$$q_{12}' = 0.8(0.174)0.624(7.6^4 - 5.4^4) = 215.5 \text{ B hr}^{-1} \text{ ft}^{-1}$$

The prime sign indicates that the result is given per unit length.

XI–11 General Equations for Heat Exchange by Radiation

All the relations used in the previous cases are similar to Eq. XI–13 except for a term which depends upon the emissivities of the materials. A form of equation for all is

$$q_{12} = F_E \sigma A_1 (T_1^4 - T_2^4) \qquad \text{[XI-18]}$$

In this relation F_E is an emissivity factor which shows the deviation from black body conditions in so far as the emissivities are concerned.

For many cases it is necessary to add further a configuration factor or area factor F_A which accounts for the fact that not every point of the one area can be connected with every point of the other area by a

straight line which does not intercept either of the surfaces. This situation is commonly stated by saying that the two radiating surfaces do not see each other completely. Thus the general equation becomes

$$T_1^4 - T_2^4)$$

Consider the two radiating areas dA_1 and dA_2 in Fig. XI-2 at temperatures T_1 and T_2. The net interchange of energy if black body conditions prevail is equal to the energy absorbed by dA_2 which represents a fraction of all the energy radiated by dA_1, minus the energy absorbed by dA_1 which represents a fraction of all the energy radiated by dA_2. It may be shown that this net interchange of radiant energy is

$$dq_{12} = \frac{\cos \quad \cdot \cos \quad \quad 1_1 \cdot dA}{\pi r^2} \times$$

$$_1^4 - T_2^4) \quad \text{[XI-20]}$$

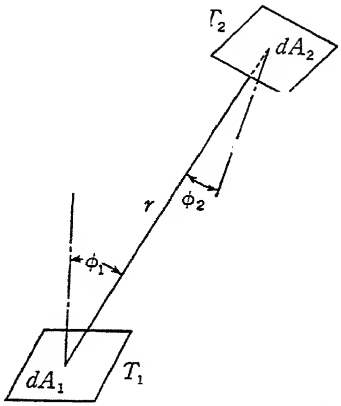

Fig. XI-2. Denotations defining the configuration of two areas.

This follows from the fact that heat radiation like light follows the law of the inverse square of the distance. The evaluation of q by integration is difficult for complex cases. It is beyond the scope of this text to show the steps in the solution of the equation. For a particular case, however, the final solution will be presented.

XI-12 Heat Exchange by Radiation Between Equal Parallel and Opposite Squares

For this case Hottel (Ref. XI-2) recommends the use of Eq. XI-19 with $F_E = \epsilon_1\epsilon_2$ when the areas are small compared to the distance apart, and with

$$\epsilon_2$$

when the areas are close together. By integration of Eq. XI-20 Hottel has found values of F_A as given in Table XI-4 where F_A is represented as a function of the ratio of the square side S to the distance D.

TABLE XI–4

AND OPPOSITE SQUARES

$\frac{S}{D}$	F_A
0.5	0.07
1.0	0.21
1.5	0.32
2.0	0.42
2.5	0.49
3.0	0.55
3.5	0.59
4.0	0.64
4.5	0.66
5.0	0.69
5.5	0.72
6.0	0.74

EXAMPLE XI–5. Calculate the radiation between the floor (15 ft by 15 ft) of a furnace and the roof if the two areas are located 10 ft apart. The floor and roof temperatures are 2000 F and 600 F respectively. Assume that the emissivity values for the floor and roof are 0.9 and 0.8 respectively.

Solution:

$$A_1 = A_2 = 15(15) = 225 \text{ ft}^2$$

$$T_1 = 460 + 2000 = 2460 \text{ R} \quad \text{and} \quad T_2 = 460 + 600 = 1060 \text{ R}$$

$$\epsilon_1\epsilon_2 = 0.9(0.8) = 0.72$$

$$\frac{1}{\dfrac{1}{\epsilon_1} + \dfrac{1}{\epsilon_2} - 1} = 0.735$$

Take $F_E = 0.73$. $S/D = 15/10 = 1.5$. From Table XI–4, $F_A = 0.32$. With these values, Eq. XI–19 becomes

$$q_{12} = (0.17)0.73(0.32)225(24.6^4 - 10.6^4)$$
$$= 3{,}240{,}000 \text{ B/hr}$$

PROBLEMS

XI–1. If 360 British thermal units of radiant energy strike a body having an absorptivity of ½ and a reflectivity of ¼, calculate the heat transmitted through the body.

XI–2. Calculate the radiant heat loss passing through a 2-in. diameter opening in a furnace door, if the furnace and outside temperatures are 1800 F and 70 F respectively. Assume black body conditions.

XI–3. Calculate the radiant heat loss from a 3-in. nominal wrought iron oxidized pipe (outside diameter = 3.5 in.) at 200 F which passes through a room the temperature of which is maintained at 76 F.

XI–4. If the heat transfer by radiation from the unpainted side of a house wall to the plaster side is 10 B hr^{-1} ft^{-2}, calculate the emissivity of the unpainted wall. Assume that the average surface temperatures of the unpainted side and the plastered side are 110 F and 90 F respectively. Emissivity of the plastered wall equals 0.93.

XI–5. If the average rate of radiant energy received and absorbed at the earth's surface on a clear day is 300 B hr^{-1} ft^{-2}, calculate the approximate temperature of the sun. Assume that 100 B hr^{-1} ft^{-2} is absorbed by the earth's atmosphere. Assume that the radius of the sun and the distance between the sun and the earth are approximately 433,000 miles and 93,100,000 miles respectively.

XI–6. Calculate the direct radiant heat transfer between two parallel refractory surfaces 4 ft by 4 ft spaced 12 ft apart. Assume that the temperatures of the surfaces are 600 F and 1200 F respectively. The emissivity values for the low and high temperatures are 0.6 and 0.9 respectively.

XI–7. Calculate the decrease in radiant heat loss per square foot of surface area through the wall of a house if two sheets of building paper are replaced by two sheets of aluminum foil. The surface temperatures of the air layer between the aluminum foils remain the same as they were with the paper, and are equal to 80 F and 0 F respectively, and the emissivity values for building paper and aluminum foil are 0.9 and 0.08 respectively.

XI–8. The value of the area under a 27 C curve in a figure similar to Fig. XI–1 was found equal to 0.047 watts cm^{-2}. Calculate the Stefan-Boltzmann constant in terms of B, hr, ft, F abs (= Rankine) units.

XI–9. The walls of a furnace are made up of refractory lining ($\epsilon = 0.8$) separated by a 4-in. air space from an outer casing of firebrick ($\epsilon = 0.6$). Calculate the radiant interchange across the air space if the lining and refractory temperatures are 2200 F and 1200 F respectively.

XI–10. An oxidized pipe through which hot water flows at 200 F passes through a room at 70 F. Calculate the per cent reduction in radiant heat transmission based on the oxidized pipe loss, if the pipe is painted with aluminum paint. Assume that the emissivity values for the oxidized and painted surfaces are 0.79 and 0.30 respectively.

REFERENCES

XI–1. W. J. Wohlenberg, "Heat Transfer by Radiation," Research Series 75, *Engineering Experiment Station Bulletin*, Purdue University, Vol. XXIV, No. 4a, August, 1940.

XI–2. H. C. Hottel, "Radiant Heat Transmission," *Mech. Engg.*, **52**, 699 (1930).

CHAPTER XII

HEAT TRANSFER BY THE COMBINED EFFECT OF CONDUCTION, CONVECTION, AND RADIATION

XII–1 Heat Conduction in Series with Convection and Superimposed Radiation

The combination of heat transfer by conduction and convection has been dealt with in Chapter IX. The combination of these two modes of heat transfer together with radiation will now be considered. The analysis of such complex problems as the heat transfer within a furnace of a steam generating unit, where conduction, convection, and surface and gas radiation are all involved, would indeed be very difficult. However, there are a few simple methods by which the influence of radiation together with conduction and convection may be studied.

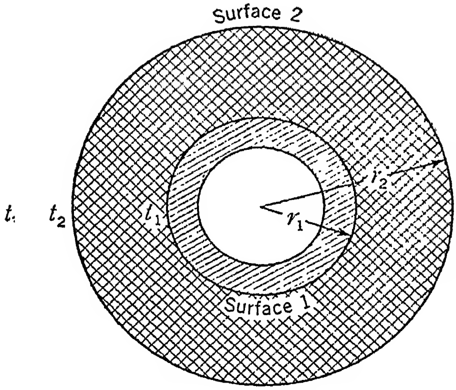

FIG. XII–1. Insulated pipe.

Consider an insulated pipe of length L and cross-sectional area as shown in Fig. XII–1. It will be assumed that the pipe is located in a room the average air and wall temperatures of which are equal to t_3. Heat is conducted radially from surface 1 to 2 according to the equation

$$q \qquad \ln r_2/r_1$$

The same amount of heat passing through the outer surface of the insulation is given up to the environment by natural convection and radiation. Fortunately radiation can be superimposed on the conductive and convective heat flow through a gas owing to its free passage through most gases. This means that the two kinds of heat transfer may be simply added as though they were like parallel electric currents.

By referring to Fig. XII–1, the rate of heat flow from the surface 2 to the air and the walls of the room is

$$q = h_c A_2 (t_2 - t_3) -$$

where h_c is the coefficient of heat transfer due to convection. The symbols in the last term on the right-hand side of the equation are the same as those used in Eq. XI–19. From Eqs. XII–1 and 2, equating and substituting the numerical value of σ, the following is obtained.

$$q = \frac{k\,2\pi L(t_1 - t_2)}{\ln r_2/r_1}$$

$$= h_c A_2(t_2 - t_3) + 0.174\, F_E F_A A_2\left[\left(\frac{T_2}{100}\right)^4 - \left(\frac{T_3}{100}\right)^4\right] \qquad \text{[XII–3]}$$

XII–2 Combined Coefficients of Convection and Radiation

It is convenient at times to replace the usual radiation equation (Eq. XI–19) by an equivalent relation of the following form:

$$q_r = h_r A_2(t_2 - t_3) \qquad \text{[XII–4]}$$

In this equation h_r is defined as a coefficient of radiation. Comparison of Eqs. XI–19 and XII–4 shows that

$$h_r = F_E F_A \sigma (T_1^2 + T_2^2)(T_1 + T_2) \qquad \text{[XII–5]}$$

Substituting q_r from Eq. XII–4 in Eq. XII–3:

$$q = \frac{k\,2\pi L(t_1 - t_2)}{\ln r_2/r_1} = h_c A_2(t_2 - t_3) + h_r A_2(t_2 - t_3)$$

Simplifying

$$q = \frac{k\,2\pi L(t_1 - t_2)}{\ln r_2/r_1} = (h_c + h_r)A_2(t_2 - t_3) \qquad \text{[XII–6]}$$

In this relation the quantity $(h_c + h_r)$ represents a combination of the convection and radiation coefficients.

Combined coefficients may be taken from Table XII–1.

TABLE XII–1

HEAT TRANSFER FROM HORIZONTAL TUBES TO STILL AIR AT ORDINARY ROOM TEMPERATURE

Combined coefficients $(h_c + h_r)$ in B hr^{-1} ft^{-2} F^{-1}

Outer Diameter D, Inches	Temperature Difference Δt[F] 50	100	150	200	250	300	350	400	450	500	550	600	650	700
1.3	2.26	2.50	2.73	3.00	3.29	3.60	3.95	4.34	4.73	5.16	5.60	6.05	6.51	6.98
3.5	2.05	2.25	2.47	2.73	3.00	3.31	3.69	4.03	4.43	4.85	5.26	5.71	6.19	6.66
5.6	1.95	2.15	2.36	2.61	2.90	3.20	3.54	3.90	4.30					
10.75	1.87	2.07	2.29	2.54	2.82	3.12	3.47	3.84	. .					

By means of this table McAdams (Ref. XII– mental data of R. H. Heilman and L. B. McMillan for pipes having oxidized surfaces. Since the emissivity of high and h_r represents only a part of the combined coefficien taken from the table may likewise be used for non-metallic su

XII–3 Heat Losses from Bare or Insulated Horizontal Tubes

In order to apply the equations used in Chapter III it was essential to know the surface temperature of the insulation. Often the temperature of a tube is known; for example, it may be assumed equal to that of the fluid inside the tube, but the temperature at the surface of the insulation is unknown. For instance, if the most suitable thickness of an insulation is to be calculated, the inside temperature of the tube and the air temperature of the room are the only known temperatures. Equation XII–6 and Table XII–1 may be used in the solution of problems of this type.

Example XII–1. Calculate the heat loss per linear foot from a 4-in. (outside diameter = 4.5 in.) nominal horizontal steel pipe covered with 1 in. of insulation (k = 0.035 B hr^{-1} ft^{-1} F^{-1}), if the pipe and still air temperatures of the room are 400 F and 70 F respectively.

Solution: The actual outer diameter of the tube is 4.5 in. and the outer diameter of the insulation is 6.5 in. For a pipe section 1 ft in length, Eq. XII–6 converts to

$$q' = \frac{q}{L} = \frac{k\,2\pi(t_1 - t_2)}{\ln r_2/r_1} = (h_c + h_r)\,2\pi r_2(t_2 - t_3) \qquad \text{[XII–7]}$$

Solving for the combined coefficient

$$h_c + h_r = \frac{k(t_1 - t_2)}{r_2(t_2 - t_3)\ln r_2/r_1}$$

Substituting the given values:

$$h_c + h_r = \frac{0.352(400 - t_2)}{(t_2 - 70)}$$

Assuming the unknown surface temperature, t_2, equal to 110 F and substituting gives,

$$h_c + h_r = 2.55 \text{ B hr}^{-1}\text{ ft}^{-2}\text{ F}^{-1}$$

For $\Delta t = 110 - 70 = 40$ F and $D = 6.5$ in., according to Table XII–1, $h_c + h_r = 1.9$. The disargeement indicates that the assumed temperature was too low.

Assume next that $t_2 = 120$ F. Then,

$$h_c + h_r = 1.97 \text{ B hr}^{-1}\text{ ft}^{-2}\text{ F}^{-1}$$

For $\Delta t = 120 - 70 = 50$, Table XII-1 yields $h_c + h_r = 1.9$. Thus the assumption that $t_2 = 120$ F is satisfactory, and from Eq. XII-7

$$q' = 1.9(2\pi)\,\frac{3.25}{12}\,(120 - 70) = 162 \text{ B hr}^{-1}\text{ ft}^{-1}$$

In the case that large temperature differences exist in the insulation, it is sometimes necessary to select new thermal conductivity values as new surface temperatures are assumed. This, however, is only a matter of routine if charts are available which indicate the thermal conductivity of the material for various temperatures.

Example XII-2. Calculate the heat loss from an uninsulated 2-in. nominal diameter (outside diameter = 2.375 in.) horizontal pipe at 400 F to the still air of a room at 70 F.

Solution: For $\Delta t = 400 - 70 = 330$ F, Table XII-1 gives $h_c + h_r = 3.67$. Therefore, the heat loss per square foot of pipe is

$$q'' = 367(400 - 70) = 1210 \text{ B hr}^{-1}\text{ ft}^{-2}$$

In the design of insulation the term insulation efficiency is often used. This is defined as the ratio of the heat saved by the insulation to the heat dissipated by the bare pipe.

Example XII-3. The heat loss from a bare pipe (1000 B hr^{-1} ft^{-1}) was reduced to 200 B hr^{-1} ft^{-1} by adding insulation. Calculate the efficiency of the insulation.

Solution: The bare pipe heat loss is 1000 B hr^{-1} ft^{-1}. The rate of heat saved by the insulation is $1000 - 200 = 800$ B hr^{-1} ft^{-1}. Therefore, the so-called insulation efficiency is $800/1000 = 0.80$ or 80 per cent.

XII-4 Heat Transfer Through Air Spaces

The combined action of thermal conduction, convection, and radiation is particularly interesting and important in air spaces. For instance, it must be considered thoroughly in the design of small frame buildings. The ideal case would exist if the convection and radiation were negligible compared with thermal conduction through the air space. So, the comparison of the effect of convection and radiation with that of conduction is of considerable practical importance.

Since free convection is governed by the laws of friction, it is obvious that the convection will be smaller in narrow spaces than in wide ones. It has been found that the convection in vertical air layers may be approximately determined by multiplying the value of the heat transfer by pure conduction with an augmentation factor of about

$1\frac{1}{4}$ for $\frac{1}{2}$-in. thickness of the layer,
$1\frac{1}{2}$ for 1-in. thickness of the layer,
$2\frac{1}{2}$ for 2-in. thickness of the layer, and
$7\frac{1}{2}$ for 5-in. thickness of the layer,

provided that the temperature difference across the layer is 10 F. The factors become larger with increasing temperature. From this it is seen how important it is for the heat insulation used in the walls and roofs of a building to have small air spaces. There is a lower limit, however, in the design, since too many narrow air layers require much more building material which has a higher thermal conductivity.

In a frame wall of given total thickness, fewer and wider air spaces mean larger temperature difference for a single layer increasing the unfavorable effect. At the same time, heat transfer by radiation likewise increases. At an average temperature of 80 F for instance and an air space 1 in. wide, the augmentation factor would be 2 instead of $1\frac{1}{2}$ for convection alone; and for a 2-in. wide space $3\frac{1}{4}$ instead of $2\frac{1}{2}$.

The influence of radiation can be reduced by using surfaces of small emissivity. Aluminum foil has been extensively used for this purpose. Even by lining only one of the radiating surfaces with aluminum foil, the influence of radiation will be greatly minimized. The effectiveness of the reflecting surfaces, however, is largely dependent upon the permanency of the surface to retain the reflective characteristic. It is claimed that the emissivity of aluminum is changed only slightly by the action of air.

Another means of decreasing the convection heat transfer is by reducing the pressure. This is done in the so-called Dewar vessels (thermos bottles), which are evacuated double-walled glass vessels. The radiation heat transfer is reduced to a very small amount by silvering the surfaces of the evacuated space.

PROBLEMS

XII–1. Determine the heat loss from a 6-in. nominal diameter (outside diameter = 6.625 in.) horizontal steam pipe 160 ft long covered with $1\frac{1}{2}$ in. of insulation ($k = 0.04$ B hr^{-1} ft^{-1} F^{-1}), if the pipe and still air temperatures are 500 F and 80 F respectively.

XII–2. Compute the heat loss per square foot of outer surface from a 4-in. nominal diameter (outside diameter = 4.5 in.) horizontal steel pipe at 400 F covered with $\frac{1}{2}$ in. of insulation ($k = 0.03$ B hr^{-1} ft^{-1} F^{-1}) to the still air of a room at 70 F.

XII–3. Calculate the heat loss per linear foot from a 4-in. nominal diameter (outside diameter = 4.5 in.) horizontal pipe covered with $1\frac{1}{2}$ in. of insulation A ($k_A = 0.056$ B hr^{-1} ft^{-1} F^{-1}) which in turn is covered with 1 in. of insulation B ($k_B = 0.039$ hr^{-1} ft^{-1} F^{-1}). The pipe surface and still air temperatures are 800 F and 80 F respectively.

XII–4. Calculate the heat loss from 200 ft of 3-in. nominal diameter (outside diameter = 3.5 in.) horizontal steel pipe at 300 F to the still air at 80 F.

XII–5. Determine the efficiency of insulation for a 6-in. nominal diameter (outside diameter = 6.625 in.) horizontal steel pipe covered with $1\frac{1}{2}$ in. of insulation ($k = 0.04$ B hr^{-1} ft^{-1} F^{-1}). The pipe and still air temperatures are 600 F and 80 F respectively.

XII-6. Compute the steam condensed per hour per linear foot of bare horizontal 6-in. nominal diameter (outside diameter = 6.625 in.) pipe, if the steam and surrounding air temperatures are 275 F and 75 F respectively. Assume that the metal temperature is the same as the steam temperature.

XII-7. Calculate the efficiency of insulation for an 8-in. nominal diameter (outside diameter = 8.625 in.) steam pipe covered with 1½ in. of insulation A next to the pipe ($k_A = 0.052$ B $\text{hr}^{-1}\,\text{ft}^{-1}\,\text{F}^{-1}$) and 2½ in. of outer insulation B ($k_B = 0.041$ B $\text{hr}^{-1}\,\text{ft}^{-1}\,\text{F}^{-1}$) if the pipe and still air temperatures are 850 F and 70 F respectively.

XII-8. Calculate a combined natural convection and radiation coefficient for a 12-in. nominal diameter (outside diameter = 12.75 in.) horizontal pipe at 300 F in contact with still air at 70 F by the laws of convection and radiation. Compare the calculated value with the one obtained by use of Table XII-1.

XII-9. Calculate the natural convection and the radiant heat losses in per cent of the total loss from a 4-in. nominal diameter (outside diameter = 4.5 in.) horizontal pipe at 200 F to the still air of a room at 70 F.

XII-10. Rework Problem XII-9 using a pipe temperature of 500 F. Compare the results for the two problems.

REFERENCE

XII-1. W. H. McAdams, *Heat Transmission*, McGraw-Hill Book Co., 1933.

CHAPTER XIII

EXPERIMENTAL DETERMINATION OF CONDUCTIVITIES AND EMISSIVITIES

XIII–1 Specific Properties of Matter

In dealing with the laws of heat transfer and insulation and their applications, various physical properties were used such as density, viscosity, specific heat, thermal conductivity, and emissivity. Whereas the first three occur in different branches of science, the last two are specific for heat transfer and insulation alone. There are numerous experimental methods by which these properties can be determined, either as to their absolute values, or relative to known values of any standard materials. To give an idea of the procedure involved in such tests, some of the simplest and more common methods will be briefly described.

XIII–2 Measurement of the Thermal Conductivity of Metals

For a determination of the thermal conductivity of a metal relative to another, two cylindrical bars of equal cross-sectional area A are soldered together end to end. One end of the composite specimen is attached to a heater and the other end to a cooling element. The cylindrical surfaces are insulated against heat losses. Then heat energy at the same rate q flows through either of the rods. The temperature drop $(\Delta t)_1$ over a length L_1 of rod 1, and $(\Delta t)_2$ for a length L_2 of rod 2 are measured by means of thermocouples. According to Eq. II–1

$$q = k_1 A \frac{(\Delta t)_1}{L_1} = k_2 A \frac{(\Delta t)_2}{L_2} \qquad \text{[XIII-1]}$$

where k_1 and k_2 are the thermal conductivities of the two materials, or

$$k_2 = k_1 \frac{(\Delta t)_1}{(\Delta t)_2} \frac{L_2}{L_1} \qquad \text{[XIII-2]}$$

No measurement of q is needed with this method. If the thermal conductivity k_1 of the metal of one rod is known, then the conductivity k_2 for the other rod may be determined from the temperature measurements and Eq. XIII–2. Practical arrangements are described for

instance in two research papers of the Bureau of Standards (Ref. XIII-1 and 2) where this method has been used for determining the thermal conductivity of various metals up to 1100 F.

XIII-3 Measurement of the Conductivity of Insulating and Building Materials

The most common method for the determination of the conductivity of insulating and building materials which are available in large plates is the so-called twin-plate method. It consists of placing a flat electrical heating plate H between two equal plates P_1 and P_2 composed of the material to be tested. These three plates are then placed between two cooling plates C_1 and C_2 (see Fig. XIII-1). The heat energy produced in H splits into two equal parts. Half of the heat flows vertically upward through plate P_1 and half downward through plate P_2. The cooling plates C_1 and C_2 usually are hollow bodies in which cooling water or other liquids pass to and fro through the sections of the cooler. Temperatures are measured by thermocouples arranged at the interfaces of H and P_1, H and P_2, P_1 and C_1, P_2 and C_2. If the plates P_1 and P_2 are equal in thickness L, area A and thermal conductivity, and if the rate of heat energy q of the heater H is known by means of electrical measurements, then according to Eq. II-1

$$\frac{q}{2} = kA\,\frac{\Delta t}{L}$$

where Δt is the temperature drop across either of the plates. If there are small differences, an average of Δt may be taken.

From the edges of plates P_1 and P_2 heat energy would be lost if they were in contact with the surrounding air. In order to avoid such losses, guard rings are used. In Fig. XIII-1 a heater H' similar in construction to heater H, surrounds the latter. Plates P_1 and P_2 are surrounded by plates P_1' and P_2' which are constructed from the same material. If by appropriate regulation of the electric heating of H', the same temperature drop across P_1' and P_2' exists as across P_1 and P_2, then no heat energy flows from H to H', or from P_1 to P_1' or from P_2 to P_2' since no temperature differences exist which might drive heat across the vertical gap. On the outer edges of H', P_1' and P_2' heat will be lost which is produced by H'. In order to reduce this as much as possible, the whole arrangement may be surrounded by a loose insulating material, such as diatomaceous earth.

Instead of arranging special guard rings, a single heater as large as H and H' and test plates as large as P_1 and P_1' may be used. In this

case, however, particular care must be taken to avoid errors; for instance, thermocouple readings close to the edge must not be used.

Materials which are on the market in cylindrical shape, such as pipe insulation (see Figs. II–4, 7, 9, and 11) are generally tested in cylindrical

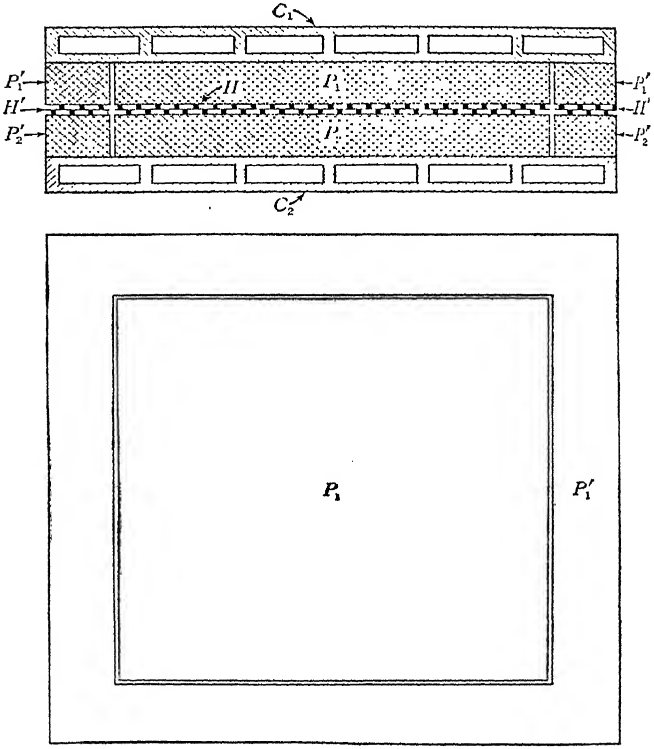

FIG. XIII–1. Twin-plate arrangement for determining the thermal conductivity of insulating and building materials.

C_1, C_2 Cooling plates
H Electrical heating plate
H' Electrical ring heater
P_1, P_2 Plates of material to be tested
P_1', P_2' Guard ring plates

form. A uniformly wound coil with a tube as a core serves as the electric heater, and a second tube is placed over this. The second tube is covered with the material to be tested. Thermocouples are arranged on the second tube and on the cylindrical outer surface of the material

to be tested. For this case, Eq. III-6 for hollow cylinders must be applied in order to determine the value of k. Heat losses at the two ends of the cylindrical sections can be avoided by guard coils on the ends which eliminate heat loss in the axial direction. They correspond to the ring heater in the twin-plate device. The end heaters may be dispensed with if long tubes are used and the thermocouples are placed at a sufficient distance from the ends.

XIII-4 Measurement of the Conductivity of Liquids and Gases

The thermal conductivity of liquids and gases likewise can be determined using either plane or cylindrical layers of the fluid. For instance, the thermal conductivity of a liquid may be determined by using an apparatus consisting of an upper heating disk and a lower cooling plate separated by a space of ⅛-in. or less which is filled with the liquid. No heat transfer by convection will take place since the density of the fluid decreases from the lower to the upper surface. By measuring the heat input, the temperatures of the disks and the vertical distance of separation between the plates, the thermal conductivity of the liquid may be obtained by means of Eq. II-1. The upward and edge heat losses may be avoided by placing a wide Dewar vessel (thermos bottle, see Sect. XII-4) over the apparatus. By this method part of the curve in Fig. II-2 was determined (Ref. XIII-3).

The co-axial cylinder method consists of arranging a fine platinum wire at the axis of a metal tube in which is placed the liquid or gas under test. The apparatus is usually placed in a liquid bath in order to regulate the surrounding temperature. The space between the inner surface of the tube and the surface of the wire forms a hollow cylinder. The platinum wire serves as an electric heater and as a resistance thermometer.* Then, referring to Eq. III-6, the rate of heat energy q' and the temperature t_1 are measured electrically. The temperature t_2 is practically equal to the temperature of the liquid bath. From these and measurements of the wire diameter $2r_1$ and the tube width $2r_2$ the thermal conductivity of the liquid or gas may be obtained from Eq. III-6. The influence of convection can be minimized by using a narrow tube, or in the case of gases, by operating at low pressure (see Sect. XII-4).

XIII-5 Measurement of Emissivities

The accurate measurement of emissivities of materials is rather difficult. Only one of the simplest methods, used at room temperature

* The principle of operation of the electric resistance thermometers is based on the well-defined relation between the change of the electric resistance of pure metals and the temperature.

(Ref. XIII–4), will be briefly described (see Fig. XIII–2). The hollow cylinder *a* with double walls for circulation of cooling water (inlet at *b*, outlet with a mercury thermometer and a thermocouple at *c*) contains a total radiation receiver *d* as shown in Fig. XIII–3. The hollow vertical copper cylinder 1 is gold plated on the outside and blackened on the inside. The two extremely thin copper disks 2 and 3 are black-

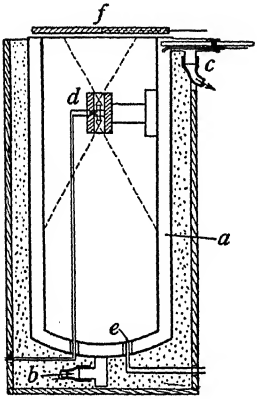

FIG. XIII–2. Device for relative measurement of emissivities at ordinary temperature.

a	Double wall container
b	Cooling water inlet
c	Cooling water outlet with thermometer and thermocouple
d	Radiation receiver
e	Thermocouple
f	Disk of material to be tested, with thermocouple in center.

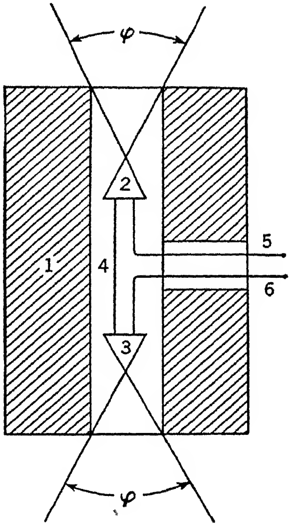

FIG. XIII–3. Radiation receiver *d* of the device represented in Fig. XIII–2.

1	Hollow copper cylinder
2, 3	Fine copper disks
4	Bismuth wire
5, 6	Wires of bismuth-tin alloy

ened on the side upon which falls the radiation from the outside. These disks which receive the radiation through the equal angles φ are heated by the incoming radiation. The warming up of the disks is measured by a system of couples consisting of a fine bismuth wire 4 and two bismuth-tin alloy wires 5 and 6. Wires 4 and 5 and 4 and 6 constitute two equal thermocouples having a thermoelectric force of 61 microvolts per degree Fahrenheit, or three times as much as an equivalent copper-

constantan system of couples. The thermocouples are connected so as to oppose each other. In this way, the thermoelectric effects produced by the absorption of equal amounts of radiation received by 2 and 3 cancel; for example, radiation from the copper cylinder 1 to the two thermocouples. Only the difference in radiation entering from above and below through the equal angles φ produces an effect.

By referring to Fig. XIII–2, it is obvious that one of the two radiations comes from the disk f, the emissivity of which is desired, and the other comes from the inner blackened surface of the container a. The surface of container a is maintained at a constant temperature which is indicated by the thermocouple e. The radiation of f at a temperature which is measured by a thermocouple indicated in the figure, is compared with that from a black body at the same temperature by replacing f with a black body. If the galvanometer to which the thermocouple system d is connected is deflected by an angle δ_b with the black body at the opening and by δ_x with the disk f at the same opening, then, aside from some corrections, the emissivity of the material of disk f which was required is $\epsilon_x = \delta_x/\delta_b$.

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* Title translated by the authors.

CHAPTER XIV

HEAT TRANSFER IN TEMPERATURE MEASUREMENTS

XIV-1 Heat Exchange, a Necessary Condition for Temperature Measurements

It is more or less an accepted notion in engineering practice that temperature measurements can be made quite accurately. This has also been implied in the former chapters of this book. This so-called trust in temperature measurements is probably based on the high standards maintained by the thermometric industry and the availability of very accurate industrial thermometric devices. Users of precision thermometric instruments often overlook the fact that accurate results are obtained only through the correct application and use of the devices, that is, the purchase, alone, of a good instrument does not assure correct temperature measurements.

Thermometry is dependent upon heat exchange. When a mercury thermometer or a thermocouple is brought in contact with a solid or fluid the temperature of which is to be measured, the instrument is supposed to assume the temperature of the solid or fluid by conduction, convection, radiation, or a combination of two or more. A radiation pyrometer operates by means of the absorbed energy emitted by a surface toward which it is directed. It is a fact that conduction, convection, and radiation occur only when a temperature difference exists. Thus, theoretically, there is always a difference between the temperature to be measured and that directly indicated by a thermometric device. It does not necessarily follow that this temperature difference will be very small.

XIV-2 Measurement of Surface Temperatures

Surface temperatures are often measured by use of a thermocouple attached to the surface. If the lead wires are arranged so that they go directly from the surface in a perpendicular direction, the system is comparable to a bar cooled in air, as discussed in Sect. IX-2. Heat from the surface whose temperature is desired is conducted to the couple contact point, and is thence conducted along the wires and finally dissipated to the surrounding air. Since a temperature difference must exist when heat flows, that part of a body which is in direct contact

with the thermocouple, will be cooler than neighboring parts of the body, and, as a result, the indicated temperature will be too low. The error involved will be larger for bodies having small conductivity values, since the heat conducted away by the lead wires will cause a greater temperature drop in the body. Thermocouple experiments by H. Reiher using plates at 96 F in air at 59 F showed that a thermocouple read 73 F if the plate used was cork, and 89 F if made of copper. When the joint of the thermocouple was arranged at the center of a thin copper disk and the disk in turn attached to the surface, the readings were 90 F for a cork plate and 94 F for a copper plate. The reason for the improvement was that the heat instead of coming from a tiny part of the surface was taken from the rather large area under the disk. When the lead wires were laid on the surface of the disk, a distance of 4 in., instead of being brought directly away from the surface the temperature indicated was 96 F for both the cork and copper plates. From these results it is apparent that large temperature measuring errors may result if thermocouples are not properly placed on the surface of insulating materials. As a general rule, surface thermocouple lead wires should always be arranged so that they lie close to the surface for a distance of a few inches in order to reduce the error due to conduction of heat along the wires.

XIV-3 Influence of Conduction and Convection on the Measurement of the Temperature of Flowing Gases

The important case of a thermometer immersed in a flowing fluid can be studied by means of the analysis given in Sect. IX-2. The upper part of Fig. XIV-1 shows a thermocouple inserted with the junction at the bottom of the well W placed in a pipe through which a gas flows at constant velocity.

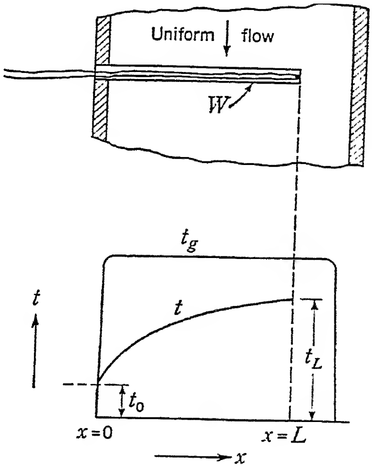

FIG. XIV-1. Temperature distribution along a thermometer well.

The temperature of the gas is assumed higher than that of the pipe. The length of the well is denoted by L, the cross-sectional area of its wall by A, and the circumference by C. The temperature distribution of the gas t_g across the pipe section is shown in the lower part of the

figure. It may be assumed that t_g is approximately constant without introducing a large error. The temperature of the well will increase from the pipe wall temperature t_0 to the temperature t_L at the end of the well. If the thermocouple wires are fine, heat conduction along them may be neglected. Radiation between the well and the pipe may likewise be negligible.

It is a matter of simple calculation to determine whether t_L is considerably lower than the gas temperature t_g which is to be measured by the thermocouple. If the well is so long that at $x = L$ no more heat will enter, then it is as though the bottom of the well were perfectly insulated. This case is identical with the one dealt with in Ex. IX–1, and solved by the formula:

$$\theta_L = \theta_0 \frac{2}{e^{mL} + e^{-mL}} \qquad \text{[IX–13]}$$

According to the definition, $\theta = t_L - t_g$ and $\theta_0 = t_0 - t_g$, both are negative. Thus, the error in the reading of the thermocouple in the well is

$$t_g - t_L = (t_g - t_0)\frac{2}{e^{mL} + e^{-mL}} = \frac{t_g - t_0}{\cosh\,(mL)} \qquad \text{[XIV–1]}$$

where $m = \sqrt{hC/kA}$, k = the thermal conductivity of the well material, and h can be calculated from Eq. VIII–5. Substituting the constants formerly given, the equation becomes

$$(Nu) = 0.3\,(Re)^{0.57} \qquad \text{[XIV–2]}$$

where $(Nu) = \dfrac{hD}{k_g}$ and $(Re) = \dfrac{vD}{\nu_g}$, and

D = the outer diameter of the well,

$$\left.\begin{aligned} v &= \text{the average velocity,}\\ k_g &= \text{the thermal conductivity,}\\ \nu_g &= \text{the kinematic viscosity}\end{aligned}\right\}\text{ of the gas.}$$

Equation XIV–1 shows that the thermometric error increases with $(t_g - t_0)$. The error becomes larger with increasing k and A and with decreasing L, h, and C, since the hyperbolic cosine of (mL) increases with mL. The error may be reduced by any one of the following means:

1. The application of insulation to the outside of the pipe wall in order to bring the wall temperature t_0 closer to the gas temperature t_g.

As a rule, the temperatures of hot gases or vapors, for example, superheated steam, should never be measured in uninsulated pipes.

2. Make the well length L as great as possible.
3. Make the cross-sectional area A of the wall as small as possible.
4. Construct the well of material having a low thermal conductivity, k.
5. Make h as large as possible. This may be accomplished chiefly by increasing the velocity. Thus, if a choice is possible the temperature of the fluid should be measured where the velocity is high. The so-called suction pyrometer (see Sect. XIV–4) is based on this idea. From Eqs. XIV–1 and 2 it may also be concluded that an increase in the diameter D of the well will improve the results slightly.

XIV–4 Influence of Convection and Radiation in the Measurement of the Temperature of Flowing Gases

In measuring the temperature of hot gases flowing through ducts the influence of radiation from the well to the cold walls must be carefully considered. In tests performed at Purdue University (Ref. XIV–1) in order to determine the errors to be expected in the measurement of flue-gas temperatures, a bare thermocouple indicated a temperature of about 775 F instead of the true gas temperature of 850 F. By placing the couple in a ⅜-in. pipe and drawing a small part of the gas through the pipe by means of a steam ejector (suction pyrometer), this error was reduced considerably.

Another method used for reducing the error is to place the thermocouple in a protecting metal shield which does not hinder the gas flow, but shields the thermocouple from radiation to the duct wall. The shield receives energy from the thermocouple by radiation and from the flowing gas by convection, and it radiates these energy amounts to the duct wall. This requires an appreciable temperature difference, and therefore the shield temperature is not much below that of the gas. Thus the thermocouple must not give up much energy by radiation and its temperature remains close to that of the gas. This means the error in the temperature reading will be small.

Consider what happens when the duct wall is not insulated and the other precautionary measures are not taken. Consider an element of the well W in Fig. XIV–1 close to the inner end where t = constant = t_L. The radiation of this element to the wall at temperature t_0 will be considered. Since the temperature of the well element is assumed almost constant, heat flow by conduction from the inner end through the wall of well W in the direction toward the pipe wall is negligible. Therefore the heat delivered by convection from the gas to the well will be given up again only by radiation to the wall of the duct and, according to

Eq. XII-2,

$$hA(t_g - t_L) = 0.174\,A\left[\left(\frac{T_L}{100}\right)^4 - \left(\frac{T_0}{100}\right)^4\right]F_E F_A \quad \text{[XIV-3]}$$

In this relation A is the surface of the element under consideration at the inner end of the well. The A terms drop out, since one appears on each side of the relation. If the duct is wide, then $F_A \approx 1$. F_E is the emissivity factor of the well surface. The coefficient of heat transfer h may be found from Eq. XIV-2.

Example XIV-1. In a 36-in. diameter circular duct a ⅝-in. diameter tube is placed at right angles to the direction of the gas flow. A thermocouple housed in the tube indicates a temperature of 800 F. If the duct wall temperature equals 500 F, the average gas velocity 15 ft/sec, and the specific weight of the gas is 0.04 lb/ft³, calculate the approximate true gas temperature. Assume that the viscosity and thermal conductivity are $0.18(10^{-9})$ lb ft⁻² hr and 0.02 B hr⁻¹ ft⁻¹ F⁻¹ respectively. The emissivity of the oxidized surface of the thermocouple housing is assumed equal to 0.8.

Solution:

$$T_L = 460 + 800 = 1260 \text{ R}$$

$$T_0 = 460 + 500 = 960 \text{ R}$$

$$\mu = 0.18(10^{-9})3600^2 \text{ slug ft}^{-1}\text{ hr}^{-1} \text{ (See Sect. VI-2)}$$

$$\rho = \frac{0.04}{32.2} \text{ slug ft}^{-3} \text{ (See Sect. I-3)}$$

$$\nu = \frac{\mu}{\rho} = 0.18(10^{-9})3600^2\,\frac{32.2}{0.04}\text{ ft}^2\text{ hr}^{-1}$$

Substituting in Eq. XIV-2:

$$\frac{h\,0.625}{0.02(12)} = 0.3\left[\frac{15(3600)\,\dfrac{0.625}{12}}{0.18(10^{-9})3600^2\,\dfrac{32.2}{0.04}}\right]^{0.57} = 0.3(1497)^{0.57}$$

$$h = 7.44 \text{ B hr}^{-1}\text{ ft}^{-2}\text{ F}^{-1}$$

Substituting this in Eq. XIV-3

$$7.44(t_g - 800) = 0.174[12.6^4 - 9.6^4]0.8$$

and

$$t_g = 1113 \text{ F}$$

This means that the thermocouple would indicate a temperature approximately 300 F too low.

XIV-5 Temperature Measurement with Radiation Pyrometers

Whereas in ordinary thermometry practice radiation is only of interest as a source of errors, high-temperature measurement is based almost entirely upon radiation. Only the main principles used in radiation pyrometry will be mentioned here.

The two types of radiation pyrometers are the optical pyrometer and the total radiation pyrometer.

Optical or selective radiation pyrometers are based on the change of intensity of radiation with temperature at a given wavelength. For black radiators this is known exactly from Planck's law (see Fig. XI-1). In this type of instrument the brightness of a glowing thread is compared with the brightness of a surface the temperature of which is to be measured. The brightness of the thread is controlled by an electric current which heats the thread until its brightness is equal to that of the surface under examination. Observing the thread by a telescope directed at the surface under consideration enables the observer to adjust the current so that the brightness of the two are equal, at which point the thread becomes invisible. A red filter is placed at the ocular of the pyrometer in order to produce monochromatic light. The device is calibrated by comparison with a black body. If the radiating surface the temperature of which is desired is not perfectly black, the true temperature may be found by use of the emissivity together with the laws of radiation.

A total radiation device has been discussed previously in the section dealing with the experimental determination of emissivities (Sect. XIII-5). The total radiation pyrometer is based on the Stefan-Boltzmann law and it consists of an optical system which is designed to collect radiant energy of all wavelengths and focus the energy on a heat-sensitive element such as a thermocouple. The thermoelectric current originating from the heating up of the one junction of the thermocouple is a measure of the total radiation which in turn is a measure of the temperature of the radiating surface. Because this type of pyrometer measures the temperature objectively, and not subjectively as the optical pyrometer, it is possible to indicate and record the reading of the instrument at any distance from the observed surface. For non-black body radiators the total radiation pyrometer is unsuited. It is most frequently used for measuring the temperature of furnaces, which may be considered as approximately black radiators.

For a complete discussion of errors, suitability of various instruments for different applications, and other information relative to temperature measurements, the reader is referred to the Power Test Codes of the American Society of Mechanical Engineers (Ref. XIV-2).

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CHAPTER XV

HEAT TRANSFER AND PRESSURE DROP

of the Relation

There are various relations between the laws of heat transmission and other physical laws. For instance, the ratio of thermal and electrical conductivity at a given temperature has the same value for all pure metals. Another example is the close analogy of mass exchange by diffusion of vapors and gases to the convective heat transfer.

In this chapter one of these analogies, which is most important for mechanical and chemical engineering, will be dealt with briefly. It concerns Osborne Reynolds' suggestion from the year 1874 (Ref. XV–1) that momentum and heat in a fluid are transferred in the same way so that in geometrically similar systems, fluid friction and heat transfer should be proportional. This analogy is of particular interest for engineers because it permits the predicting of heat transfer coefficients from friction data ,which can be obtained with less difficulty from experiments than heat transfer data. It further sets the conditions of an economical optimum in design for the following reasons.

As has been shown in Chapter VIII, heat transfer generally can be improved by increasing the speed of the fluids involved. On the other hand the pressure drop of fluids which stream in conducts increases with velocity. This means that more power is required for pumping the fluids through a heat exchanger. Therefore, the investment costs for blowers or pumps and the current costs of pumping power become greater, and eventually may offset the economies resulting from the reduction of the heating surface which is possible if higher velocities are used. Therefore, a compromise has to be found between the requirement of high velocity from the heat transfer view and the necessity of not too high velocity in order to keep the costs of overcoming the pressure drop within reasonable limits.

Some excellent presentations and modifications of Reynolds' analogy have been published recently which, however, use more mathematics and fluid mechanics than can be required in this text. Therefore, a simpler, though theoretically not so strongly founded way will be followed which has been indicated by White (Ref. XV–2).

XV–2 Basic Idea of Reynolds' Analogy

Reynolds' reasoning was, that from a molecular viewpoint fluid friction and heat conduction obey the same law of motion. If two neighboring layers in a gas moving along a plane wall have different velocity, then in the course of molecular motion, molecules of the faster layer pass to the slower one and are retarded there, whereas molecules of the slower layer come into the faster layer and are accelerated. By this interchange of molecules, the faster layer will be retarded as a whole, and the slower one will assume a greater speed. Owing to the mass of the molecules an exchange of momentum occurs which acts like friction and can be expressed by Newton's equation:

$$F_s = \mu A \frac{dv}{dy} \tag{VI–2}$$

where y is the distance from the wall of the layer under consideration.

This equation was used for the definition of the dynamic viscosity μ. According to the foregoing remarks μ may be considered as the mass per unit time and area which is interchanged by molecular action between layers unit distance apart.

This can be shown more directly from Eq. VI–2 by comparing the latter with Newton's basic law of dynamics which may be expressed in the form

$$F \qquad dv$$

where m is the mass which by the shearing or frictional force F_s is decelerated, and τ is the time.

From Eqs. VI–2 and XV–1:

$$-m \frac{\partial v}{\partial \tau} = \mu A \frac{\partial v}{\partial y} \tag{XV–2}$$

partials being used because now two independent variables (y and τ) are involved. But according to a mathematical equation about the partial differentials of an analytical function $z = f(x,y)$:

$$\frac{\partial z}{\partial x} \frac{\partial x}{\partial y} \frac{\partial y}{\partial z} = -1 \tag{XV–3}$$

In our case v is a function of y and τ. Therefore, corresponding to Eq. XV–3

$$\frac{\partial v}{\partial \tau} \frac{\partial \tau}{\partial y} \frac{\partial y}{\partial v} = -1 \tag{XV–4}$$

and by substitution in Eq. XV–2

$$\partial\tau$$

or

$$\mu = \frac{m}{A}\frac{\partial y}{\partial\tau} \qquad \text{[XV–5]}$$

For unit length and time interval, $\partial y/\partial\tau = 1$, and unit area means $A = 1$. Then $\mu = m$, that is, the viscosity equals the decelerated mass as has been asserted.

If not only velocity differences, but also temperature differences exist between the two layers under consideration, the molecules will not only exchange momentum, but will likewise convey heat energy from the warmer to the colder layer. The heat transportation which corresponds to the product $\mu \cdot dv$ in Eq. VI–2 obviously should be proportional to the interchanged mass μ, the specific heat c_p of the fluid and to the temperature difference dt. Thus one arrives at

$$q = \mu c_p A \frac{dt}{dy} \qquad \text{[XV–6]}$$

as an equation corresponding to Eq. VI–2.

However, according to the basic equation of heat conduction

$$q = -kA \frac{dt}{dy} \qquad \text{[XV–7]}$$

The difference in the signs in Eqs. XV–6 and 7 does not matter because it is due to a difference in definition only. Equation XV–6 is derived from Eq. VI–2 where $F_s > 0$ means transportation of momentum in the direction to the wall, whereas in Eq. XV–7 $q > 0$ means transportation of heat in the direction from the wall.

Thus, comparison of Eqs. XV–6 and 7 shows that Reynolds' analogy holds only if $\mu c_p = k$, or

$$\frac{\mu c_p}{k} = 1 \qquad \text{[XV–8]}$$

The dimensionless group $\mu c_p/k$ occurs in all kinds of convection. It has been called Prandtl number and denoted by the symbol (Pr) in Sect. VI–7.

As mentioned in Sect. VIII–2, (Pr) does not change much for gases, and with an average value of 0.8, is not far from the value 1. Because further $(Pr)^n$ usually appears in the equations of heat transfer where $n < 1$, Reynolds' analogy will hold for gases with good approximation.

It must be kept in mind, however, that flow in layers, i.e., streamline flow was assumed in the approximate derivation given above.

XV-3 Generalization of Reynolds' Concept

Reynolds' analogy of heat transfer and fluid friction in its original form is not only restricted to gases, more exactly to a fluid with $(Pr) = 1$, but also to streamline flow. In order to get rid of the latter restriction a fictitious or apparent viscosity μ_a will be introduced which includes the exchange of momentum by mixing of larger aggregates of molecules as occurs in turbulence, instead of the merely molecular mixing. In making use of μ_a, one must keep well in mind that this is a magnitude which depends upon the motion and so may vary from point to point in the stream whereas the molecular or true viscosity μ is a property of the fluid which in a rather wide pressure range depends on the temperature only and nothing else. So μ_a may be almost equal to μ at one point of the fluid cross section, and 100 times larger than μ at another point.

In a similar way, instead of the thermal conductivity k, a fictitious or apparent conductivity k_a may be introduced, which takes care of the exchange of heat energy between the mentioned larger aggregates of molecules. This magnitude is likewise very different from the true thermal conductivity, nor can it be assumed that $\mu_a/k_a = \mu/k$ or $\mu_a c_p/k_a = \mu c_p/k$. Considering for instance an aggregate of molecules which moves from one layer to another one and then returns to the first one, momentum may be exchanged so rapidly that the exchange is completed when the aggregate returns to the original layer, but the interchange of heat energy may not be completed in this time interval. Then $\mu_a = \mu$, but $k_a < k$, and by this $\mu_a c_p/k_a > \mu c_p/k$. For this and other reasons it is not at all certain *a priori* that Reynolds' analogy can be used in turbulent flow.

A comparison between the experimentally observed distributions of temperature and velocity has been considered as a simple test of the theory. If the errors due to the made approximations are unimportant, then the two distributions in any plane normal to the flow direction should be similar.

The first to perform such experiments was Pannell (Ref. XV-3). He measured the velocity and temperature distribution across air which streamed in a vertical brass tube, 1.92-in. inside diameter. The velocity and temperature in the center line of the tube were $v_c = 87.4$ ft/sec, and $t_c = 75.9$ F respectively, the wall temperature was $t_w = 109.4$ F. The test cross section was far removed from the pipe entrance so that the effect of entrance disturbances was greatly diminished.

The air velocity and temperature at a radial distance y from the tube wall may be called v and t respectively. Introducing further $\theta = t_w - t$, the velocity ratio v/v_c can be compared with the temperature ratio θ/θ_c for different values of the distance ratio y/r, where r is the distance of the center line from the wall, and $\theta_c = t_w - t_c$.

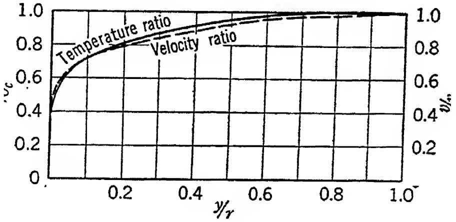

FIG. XV–1. Distribution of velocity and temperature across an air stream in a heated tube, according to measurements of J. R. Pannell, in dimensionless representation.

In this way Pannell's main results are represented in Fig. XV–1 and Table XV–1. Similarity obviously exists if $v/v_c = \theta/\theta_c$ for any radial distance, and the ratios shown in the figure and the table indeed correspond to this condition with good approximation.

TABLE XV–1

VELOCITY AND TEMPERATURE RATIO FOR TURBULENT FLOW OF AIR IN A TUBE, ACCORDING TO MEASUREMENTS OF J. R. PANNELL

$\frac{y}{r}$	$\frac{v}{v_c}$	$\frac{\theta}{\theta_c}$
0	0	0
0.05	0.66	0.65
0.1	0.73	0.73
0.2	0.80	0.81
0.4	0.89	0.92
0.6	0.96	0.98
0.8	0.99	1.00
1.0	1.00	1.00

White (Ref. XV–2) has pointed to the strange fact that this agreement is much better than for streamline motion for which Reynolds' analogy was derived originally. From this it is seen that Reynolds' theory does not hold exactly. The main reasons for the deviations seem to be a certain lack in the assumed similarity of heat transfer and interchange of momentum in a tube. This, however, cannot be dealt with here in detail.

XV-4 Equations Based on Reynolds' Analogy

In order to calculate the heat flow from the wall to the fluid it is necessary to know the temperature slope $d\theta/dy$ in the fluid adjacent to the wall. This is very steep for turbulent flow as can be seen from Fig. XV-1 where the curves begin almost vertically at the left-hand side. Fortunately, the slope of the curves can be expressed in terms of heat flow and fluid friction.

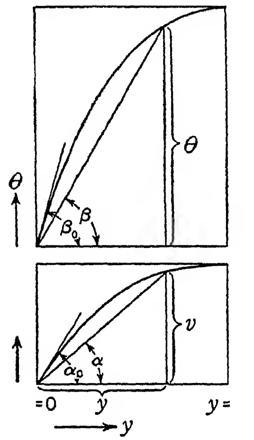

FIG. XV-2. Schematic representation of velocity and temperature distribution across a fluid stream in a heated tube. (Since $\theta = t_w - t$, the temperature t is great where θ is small, and vice versa.)

Figure XV-2 is a schematic representation of similar velocity and temperature distributions across the fluid stream in a tube. The similarity of the two curves can be checked by comparing the angles between the horizontal axis and straight lines from the zero point to points of the two curves at the same distance y. Let these angles be α and β respectively. It is seen that they increase with decreasing v and θ up to maximum values of α_0 and β_0, determined by the tangents of the curves at the zero point of the system.

Obviously, a condition of similarity is, that for any distance y between $y = 0$ and $y = r$,

$$\frac{\tan \alpha}{\tan \alpha_0} \quad \frac{\tan \beta}{\tan \beta_0} \qquad \text{[XV-9]}$$

The four tangent terms may be expressed by the pertinent geometrical and physical data.

From Fig. XV-2 it is seen that

$$\tan \alpha = \frac{v}{y} \qquad \text{[XV-10]}$$

$$\tan \beta = \frac{\theta}{y} \qquad \text{[XV-11]}$$

According to Eq. VI-2, the frictional force on the wall is

$$F_w = \mu A \left(\frac{dv}{dy}\right)_{y=0} = \mu A \cdot \tan \alpha_0 \qquad \text{[XV-12]}$$

According to Eq. XV-7, the rate of heat flow from the wall to the fluid is

$$q_w = -kA\left(\frac{dt}{dy}\right)_{y=0}$$

Substituting $t = t_w - \theta$, according to the definition of θ, and considering that the wall temperature t_w is supposed to be constant, it follows that

$$q_w = kA\left(\frac{d\theta}{dy}\right)_{y=0} = kA \cdot \tan\beta_0 \qquad [XV-13]$$

By substitution from Eqs. XV-10, 11, 12, and 13 in Eq. XV-9 one obtains

$$\frac{q_w y}{\theta k} = \frac{F_w y}{\mu v}$$

or

$$\frac{q_w v}{F_w \theta} = \frac{k}{\mu} = \frac{kc_p}{\mu c_p} = \frac{c_p}{(Pr)}$$

or

$$\frac{q_w v\,(Pr)}{F_w \theta c_p} = 1 \qquad [XV-14]$$

This, however, has been derived under the assumption of laminar flow. For turbulent motion it would hold only if $\mu_a c_p/k_a = \mu c_p/k$ which is unlikely. Generally, $\mu_a c_p/k_a$ will be greater than $\mu c_p/k$, as has been explained in Sect. XV-3. Furthermore, the lack in similarity, mentioned at the end of that section, must be considered.

For these reasons White recommended the use of the following modified equation:

$$\frac{q_w v_m\,(Pr)}{F_w \theta_m c_p} = \phi \qquad [XV-15]$$

where v_m and θ_m are mean values, $t_m = t_w - \theta_m$ being the mixing-cup temperature as defined in Sect. VIII-1.

The exact theory for streamline flow and uniform heating from the wall shows that

$$\phi = 0.545 \qquad [XV-16]^*$$

* According to Eqs. XV-18 and 19, $F_w = \dfrac{16}{(Re)} A\rho \dfrac{v_m^2}{2}$. If the tube is uniformly heated from outside, $q_w = 2.18(k/r)\,A\theta_m$, as derived first by Callendar (Ref. XV-4). Substituting these two expressions in Eq. XV-15, one obtains Eq. XV-16.

For turbulent flow, White found the following empirical relation:

$$\phi = \frac{\sqrt{(Pr)}}{1 + \dfrac{750\sqrt{(Pr)}}{(Re)} + \dfrac{7.5\sqrt[4]{(Pr)}}{\sqrt{(Re)}}} \qquad \text{[XV-17]}$$

This gives rather satisfactory results for $Re \geqq 6000$ and $Pr \lesseqgtr 50$. If the application of Eq. XV–16 is restricted to quite moderate temperature differences, θ_m, the calculation of (Re) and (Pr) may be based on the average temperature of the flowing fluid. The more complicated procedure for larger values of θ_m will not be dealt with in this book.

The effect of disturbances at the entrance of the tube is not included in Eq. XV–17; it will be considered approximately in Sect. XV–5.

In fluid dynamics it is usual to express the frictional force F_w, which is exerted on the inner surface of a tube, in terms of the kinetic energy per unit volume. This is done by introduction of a dimensionless coefficient of friction, often called Fanning's factor:

$$f = \frac{F_w/A}{\rho \dfrac{v_m^2}{2}} = \frac{F_w/A}{\dfrac{\gamma}{g}\dfrac{v_m^2}{2}} \qquad \text{[XV-18]}$$

A, in this equation, is the area of the inner surface; ρ and γ are the density and specific weight, respectively, as defined in Sect. I–3.

The exact theory of streamline flow leads to

$$f = \frac{16}{(Re)} \qquad \text{[XV-19]}$$

For turbulent flow the empirical equation

$$f = \frac{0.08}{\sqrt[4]{(Re)}} \qquad \text{[XV-20]}$$

holds at least up to $(Re) = 200{,}000$.

Solving now Eq. XV–15 for q_w, and substituting F_w from Eq. XV–18 one obtains

$$q_w = \phi f \frac{\gamma c_p}{2g(Pr)} A v_m \theta_m = \phi f \frac{C_p}{2(Pr)} A v_m \theta_m \qquad \text{[XV-21]*}$$

An average film coefficient of heat transfer h_m is defined in the usual manner by the formula

$$q_w = h_m A \theta_m \qquad \text{[XV-22]}$$

* According to Sect. IV–1: $C_p = \rho c_p = \frac{\gamma}{g} c_p$.

From Eqs. XV–21 and 22:

$$h_m = \phi f \frac{\gamma c_p}{2g(Pr)} v_m = \phi f \frac{C_p}{2(Pr)} v_m \qquad \text{[XV–23]}$$

The right side of this equation contains only the diameter of the tube, the velocity, and some properties of the flowing substance. Thus, the coefficient of heat transfer is determined without reference to heat flow or temperature.

In engineering, it is more usual to operate with the pressure drop Δp in a pipe than with the frictional force F_w exerted on the inner surface. Δp may be introduced by means of the following relation which is used generally in fluid dynamics:

$$\Delta p = 4f \frac{L}{D} \frac{\rho v_m^2}{2} = 4f \frac{L}{D} \frac{\gamma v_m^2}{2g} \qquad \text{[XV–24]}$$

L and D in this equation are the length and inner diameter of the pipe, respectively. It is seen that expressing lengths in feet and γ in lb/cu ft, Eq. XV–24 yields Δp in lb/sq ft.

Substitution of f from this equation in Eq. XV–23 leads to

$$h_m = \frac{\phi}{4} \frac{c_p}{(Pr)} \frac{D}{L} \frac{\Delta p}{v_m} \qquad \text{[XV–25]}$$

for both streamline and turbulent flow.

According to Eq. XV–19, f is reciprocal to (Re) and by this to v_m for streamline flow. Then it follows from Eq. XV–23 that h_m is independent of the velocity.

For turbulent flow, according to Eqs. XV–20 and 23, h is about proportional to $v_m^{3/4}$. There is still another influence of velocity, because, according to Eq. XV–17, also the factor ϕ depends on (Re). But this influence is small for great values of (Re), and for small values of (Pr). Water at 100 F, for instance, has $(Pr) \approx 4$. With this value, Eq. XV–17 yields $\phi = 1.83$ for $(Re) = 40{,}000$, and $\phi = 1.77$ for $(Re) = 90{,}000$, so that the change in ϕ is almost negligible.

Example XV–1. Air at an average absolute pressure of 29.4 lb/sq in. and an average temperature of 70 F is flowing through a pipe line, 1000 ft long, 3-in. inner diameter, its average velocity being 30 ft/sec.

Thermocouples attached to the wall show an average temperature of 60 F. Assuming the inner surface to be smooth, and neglecting entrance flow disturbances, find the pressure drop, the heat loss per hour of the air, and the film coefficient of heat transfer on the inner pipe wall.

Solution: The mean temperature difference is $\theta_m = t_m - t_w = 70 - 60 = 10$ F. This is so small that all properties of the air may be based on the average temperature $t_m = 70$ F.

The first thing to be found out is, whether the flow is streamline or turbulent.

$$(Re) = \frac{v_m D \rho}{\mu}$$

$$v_m = 30(3600) = 108{,}000 \text{ ft/hr}$$

$$D = 3/12 = 0.25 \text{ ft}$$

$$\gamma = 0.15 \text{ lb/cu ft (at 29.4 lb/sq in.)}$$

$$\rho = 0.15/32.2 = 0.00466 \text{ slug/cu ft}$$

Because the pressure does not influence much the dynamic viscosity, μ can be taken from Table VI–2.

$$\mu = 0.105(10^{-9})3600^2 = 0.00136 \text{ slug ft}^{-1} \text{ hr}^{-1}$$

$$(Re) = \frac{108{,}000(0.25)\ 0.00466}{0.00136} = 92{,}500$$

Thus, the flow is turbulent, and in calculating Δp from Eq. XV–24, the friction factor f may be taken from Eq. XV–20.

$$f = \frac{0.08}{\sqrt[4]{92{,}500}} = 0.0046$$

$$\Delta p = 4(0.0046)\frac{1000}{0.25}\frac{0.00466(30)^2}{2} = 154 \text{ lb/sq ft} = 1.07 \text{ lb/sq in.}$$

The heat loss may be found either from Eq. XV–25 or from Eq. XV–21, with ϕ to be calculated by means of Eq. XV–17.

$$(Pr) = \frac{\mu c_p}{k}$$

$$c_p = 0.24(32.2) = 7.73 \text{ B slug}^{-1} \text{ F}^{-1}$$

$$k = 0.0148 \text{ B hr}^{-1} \text{ ft}^{-1} \text{ F}^{-1} \text{ (from Fig. II–1)}$$

$$(Pr) = \frac{0.00136(7.73)}{0.0148} = 0.71$$

$$C_p = (0.15)\ (0.24) = 0.036 \text{ B ft}^{-3} \text{ F}^{-1} \text{ (see Sect. IV–1)}$$

$$\phi = \frac{\sqrt{0.71}}{1 + \frac{750\sqrt{0.71}}{92{,}500} + \frac{7.5\sqrt[4]{0.71}}{\sqrt{92{,}500}}} = \frac{0.843}{1.029} = 0.82$$

$$A = L\pi D = 1000\ \pi 0.25 = 785 \text{ sq ft}$$

Substituting all these terms in Eq. XV-21:

$$q_w = 0.82(0.0046)\,\frac{0.036}{2(0.71)}\,785(30)10 = 22.5 \text{ B/sec}$$

or

$$q_w = 22.5(3600) = 81{,}000 \text{ B/hr}$$

The film coefficient of heat transfer is found from Eq. XV-22.

$$h_m = \frac{81{,}000}{785(10)} = 10.3 \text{ B hr}^{-1}\text{ ft}^{-2}\text{ F}^{-1}$$

This result may be checked without using the pressure drop analogy, by means of Eq. VIII-2. Substituting the pertinent values in this equation:

$$h_m = \frac{0.0148}{0.25}\,0.023(92{,}500)^{0.8}\,(0.71)^{0.4} = 11.1 \text{ B hr}^{-1}\text{ ft}^{-2}\text{ F}^{-1}$$

The agreement of the results, found in such different ways and using partly empirical equations and some simplifications, is satisfactory.

It is seen that Eq. VIII-2 leads much easier to the result than the analogy method. If, however, Eq. VIII-2 were not known, or in other cases where direct heat transfer measurements were missing and were difficult to perform, the indirect method would be very useful. If, for instance, the tubes of a heat exchanger are curved, the influence of curvature to heat transfer can be calculated from the results of pressure drop measurements as will be shown in the next section.

XV-5 Secondary Influences

The equations of Sect. XV-4 are based on the assumptions that the tubes under consideration are perfectly smooth and straight, that no entrance disturbances exist, and that the temperature difference θ_m is small. In practice, there may be deviations from each of these conditions.

Deviations from the first and last of them will not be dealt with here. However, ordinary drawn tubes can be considered as smooth, and the equations give good approximations also for moderately rough tubes and for moderate temperature differences.

The local disturbances of the flow near the entrance of a tube give rise to an additional pressure drop which, according to White, can be considered by adding

$$\delta(f) = 0.1\,\frac{D}{L} \qquad \text{[XV-26]}$$

to the value of f, as found by Eq. XV-20. This is said to hold for abrupt entrances.

At $R = 10{,}000$, $f = 0.008$. If $L/D = 12.5$, Eq. XV–26 yields $\delta(f) = 0.008$. This means that under the above circumstances the entrance loss is just as great as the ordinary loss in an additional length $\delta(L) = 12.5\ D$.

In calculating the heat transfer the same additions may be provided although there is not much experimental evidence about this.

Concerning the influence of curvature of tubes, White's experiments on the turbulent flow resistance of closely coiled helixes led to:

$$f = \frac{0.08}{\sqrt[4]{Re}} + 0.012\sqrt{\frac{D}{C}} \qquad \text{[XV–27]}$$

where D = the inner pipe diameter,
C = the mean coil diameter.

This holds from $Re = 15{,}000$ to $Re = 100{,}000$.

The heat transfer in this range may be calculated from Eq. XV–15, using Eq. XV–27 instead of Eq. XV–20.

XV–6 Heat Transfer and Fluid Friction on Plane Surfaces

In a fluid flowing along a plate, the length L of the plate may be used as characteristic length so that

$$(Re) = \frac{v_m L \rho}{\mu} \qquad \text{[XV–28]}$$

Colburn (Ref. XV–5), in a correlation of experimental data, distinguishes between a streamline or viscous region up to $(Re) \approx 20{,}000$ and a turbulent region above this.

Using the definition of f, given by Eq. XV–18, he shows that for a streamline flow:

$$f = \frac{1.32}{\sqrt{(Re)}} \qquad \text{[XV–29]}$$

and for turbulent flow:

$$f = \frac{0.072}{\sqrt[5]{(Re)}} \qquad \text{[XV–30]}$$

These equations correspond to Eqs. XV–19 and 20 for tubes.

For both, viscous and turbulent range, Colburn further recommends the formula

$$h_m = f\frac{\gamma c_p}{2\,g\,(Pr)^{2/3}}\,v_m = f\frac{C_p}{2(Pr)^{2/3}}\,v_m \qquad \text{[XV–31]}$$

which corresponds to Eq. XV–23.

In the flow of fluids across tubes, as in tube banks, at the front and rear of the obstacle, forces originate which have nothing to do with the exchange of momentum. Therefore, the simple relation between surface friction and heat transfer fails. Concerning this, the reader is referred to the quoted papers of White (Ref. XV-2) and Colburn (Ref. XV-5).

PROBLEMS

XV-1. Sulfuric acid is pressed through a capillary tube, ⅛ in. wide and 10 ft long, with well-rounded entrance opening. The pressure difference between the inlet and outlet is 10 lb/sq in., the mean temperature of the fluid is 40 F. How much heat energy per degree Fahrenheit difference between wall temperature and mean liquid temperature will the acid be able to pick up in one hour?

The density of the acid is 1.84 gram/cu cm, the dynamic viscosity is 45 centipoise, the thermal conductivity is 0.145 B $hr^{-1}\,ft^{-1}\,F^{-1}$, and the Prandtl number is 240.

XV-2. Water at 60 F average temperature flows through a straight piece of drawn copper tubing, 1 in. diameter, 20 ft long, and then through a coil of the same pipe, having twenty turns of 15-in. mean diameter, which is located in a liquid bath of constant temperature. The pressure drop in the straight piece of pipe is found to be equal to ½-in. mercury column, and the wall temperature of the coil is held at the constant temperature of 80 F. Calculate the coefficient of heat transfer at the inside surface of the coil (*a*) using the analogy between surface friction and heat transfer and (*b*) without using this analogy.

XV-3. Air at standard atmospheric pressure and 50 F average temperature streams along a plane plate, 5 ft wide, with an average velocity of 50 ft/sec. A length of 15 ft of this plate is held at 100 F by a heating device. Neglecting all secondary influences, like that of the edges of the plate or of the increase of temperature of the air along the plate, calculate how much heat per hour is given up to the streaming air.

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INDEX

B